INVITED CONFERENCE
Shear and punching

Aurelio Muttoni

EUROCODES

EN 1992

Design of concrete structures

2nd generation of Eurocode 2 on concrete structures

Madrid, October 17th, 2023

Documento descargado de www.e-ache.com el 05/11/2026

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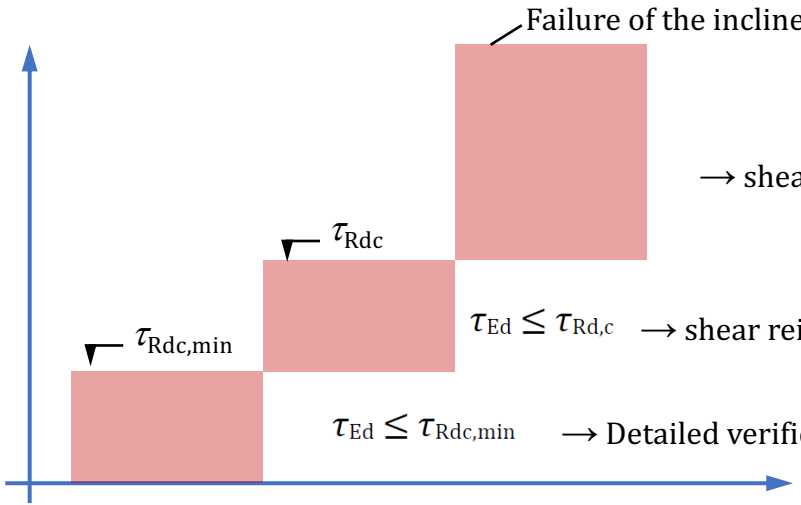
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Shear, verification procedure

$$\tau_{Ed} = \frac{V_{Ed}}{b_w \cdot z}$$ for linear members

or

$$\tau_{Ed} = \frac{v_{Ed}}{z}$$ for planar members



$\longrightarrow$ shear reinforcement shall be designed

$\tau_{Ed} \leq \tau_{Rd,c}$ $\longrightarrow$ shear reinforcement is not required

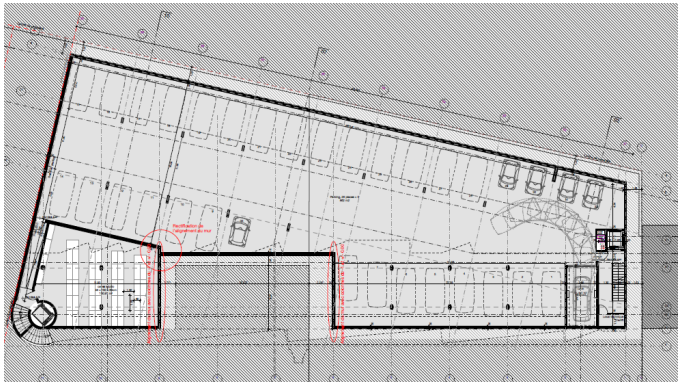
$\tau_{Ed} \leq \tau_{Rdc,min}$ $\longrightarrow$ Detailed verification of the shear resistance may be omitted

z is the lever arm for the shear stress calculation defined as $z = 0{,}9d$

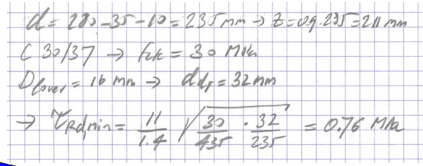
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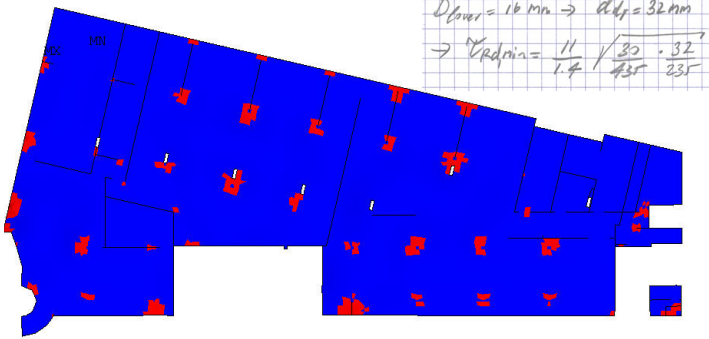
Verification of the minimum shear stress resistance

$$\tau_{\mathrm{Rdc,min}} = \frac{11}{\gamma_{\mathrm{V}}} \cdot \sqrt{\frac{f_{\mathrm{ck}}}{f_{\mathrm{yd}}} \cdot \frac{d_{\mathrm{dg}}}{d}}$$



$d_{\mathrm{dg}} = 16\ \mathrm{mm} + D_{\mathrm{lower}} \leq 40\ \mathrm{mm}$ for ($f_{\mathrm{ck}} \leq 60$ MPa) or
$d_{\mathrm{dg}} = 16\ \mathrm{mm} + D_{\mathrm{lower}}\,(60/f_{\mathrm{ck}})^2 \leq 40\ \mathrm{mm}$ for ($f_{\mathrm{ck}} > 60$ MPa)





- Simple verification without knowing the flexural reinforcement possible
- Areas in slabs where a shear verification is required can be easily detected (refined verification for shear or punching required in governing areas only)

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Refined verification of the shear stress resistance (members without shear reinforcement)

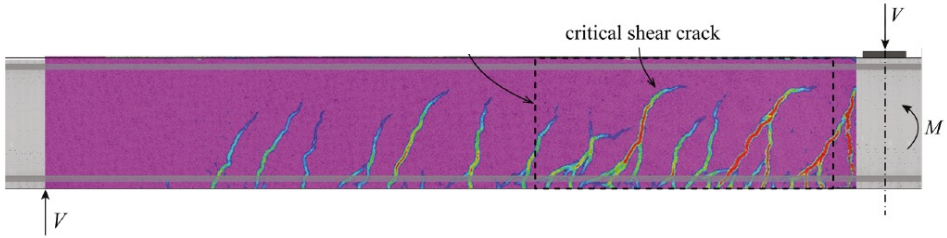
$$\tau_{Rd,c} = \frac{0{,}66}{\gamma_V} \cdot \left(100\rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{d}\right)^{\frac{1}{3}} \geq \tau_{Rdc,min}$$

- Formula looks similar as in current EN 1992-1-1:2004
- What is new?

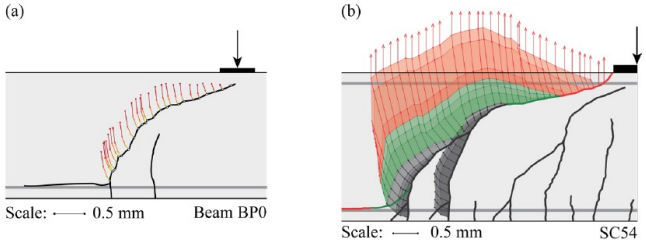
- Formula derived analytically from the general expression of the Critical Shear Crack Theory (formula in current EN 1992-1-1:2004 was fully empirical)
- Allows for several levels of refinement
- Can be easily generalized to others cases (effect of axial force, different types of concrete, non-metallic reinforcement, …)
- Recommended value γ_V = 1,40 due to the fact that the variability of f_{ck} has a smaller influence on the shear resistance, but higher model uncertainties than for compression members used to calibrate γ_C = 1,50

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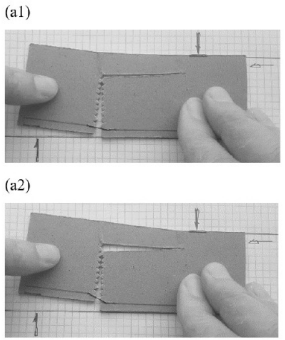
Bases of the Critical Shear Crack Theory (instance of shear in members without shear reinf.)



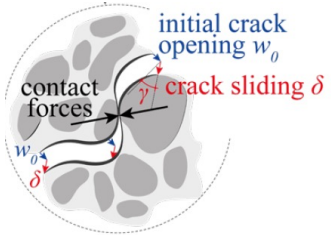
Adapted from Cavagnis et al. 2018



Measurement results by Muttoni 1985 and Cavagnis 2015



Original cardboard model by Muttoni, 1985 (see Muttoni and Simões, 2023)

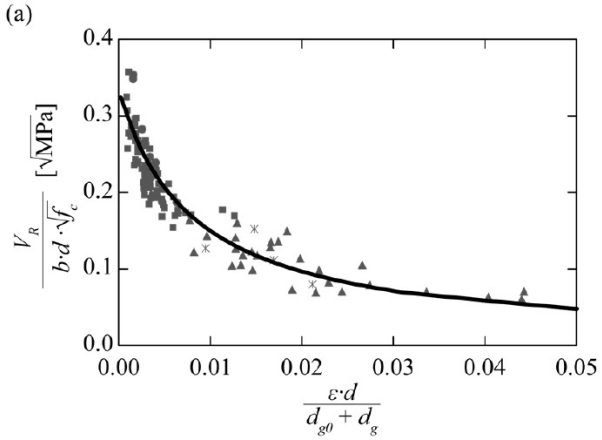


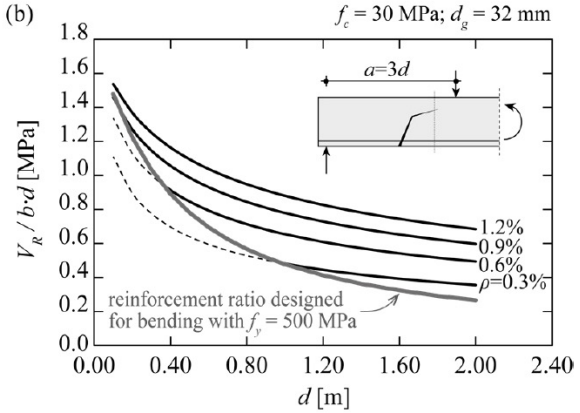
⇒ The shear resistance depends on:

- Strain in the longitudinal reinforcement
- Effective depth
- Roughness of the Critical Shear Crack (which depends mainly on the aggregate size)

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Bases of the Critical Shear Crack Theory (instance of shear in members without shear reinf.)



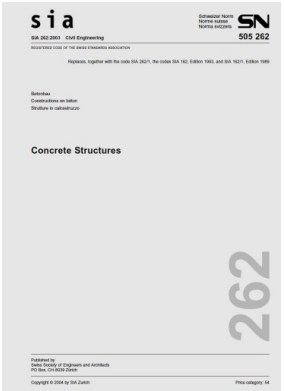


Adapted from Muttoni 2003

$$V_{Rd} = \frac{0.3}{1 + \varepsilon_v \cdot d \cdot k_{dg}} \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d$$

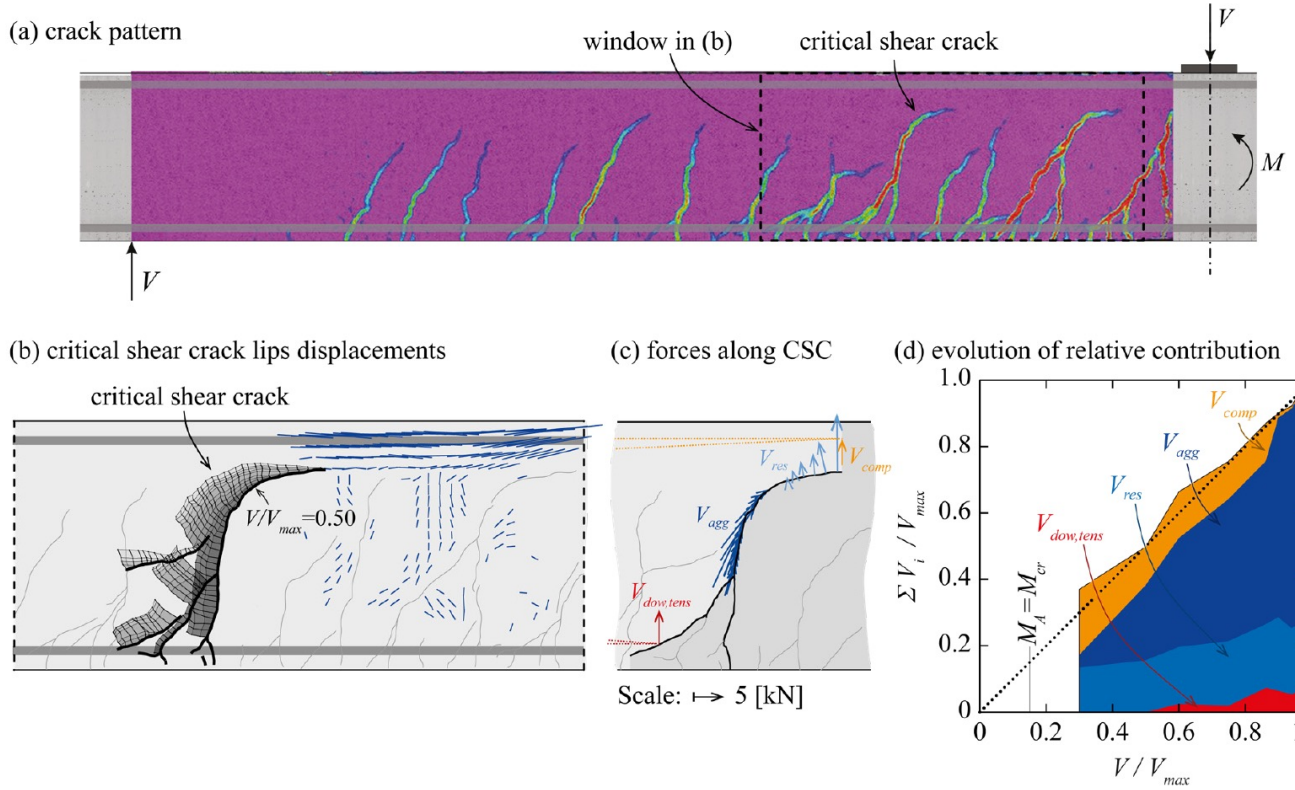
$$\varepsilon_v = \frac{f_{yd}}{E_s} \frac{M_{Ed}}{M_{Rd}}$$

Swiss Code for concrete structures, 2003



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Bases of the Critical Shear Crack Theory (instance of shear in members without shear reinf.)

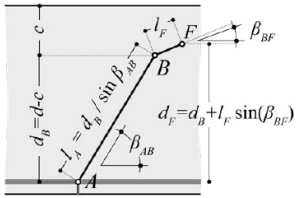


Adapted from Cavagnis et al. 2018

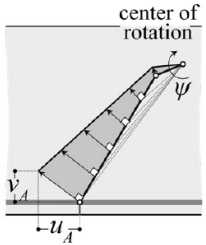
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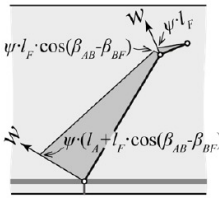
(b) CSC geometry



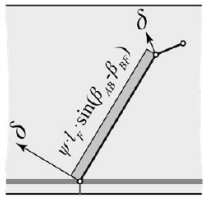
(c) CSC kinematics



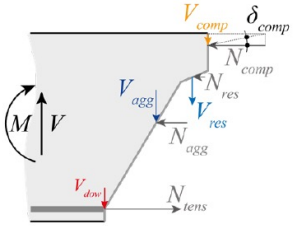
(d) crack opening



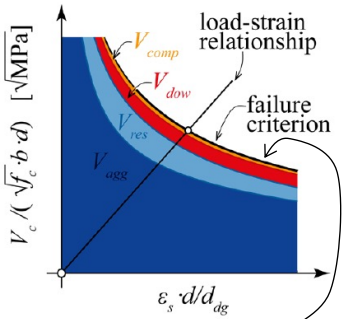
(e) crack sliding



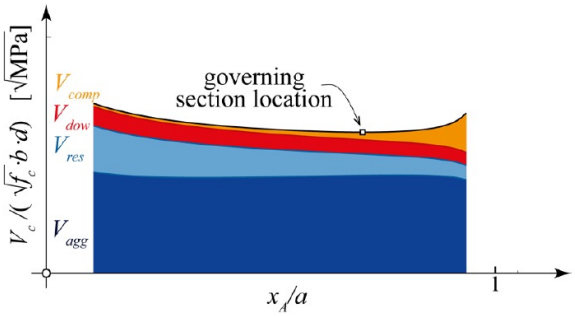
(f) equilibrium



(g) failure load for given section



(h) determination of governing section

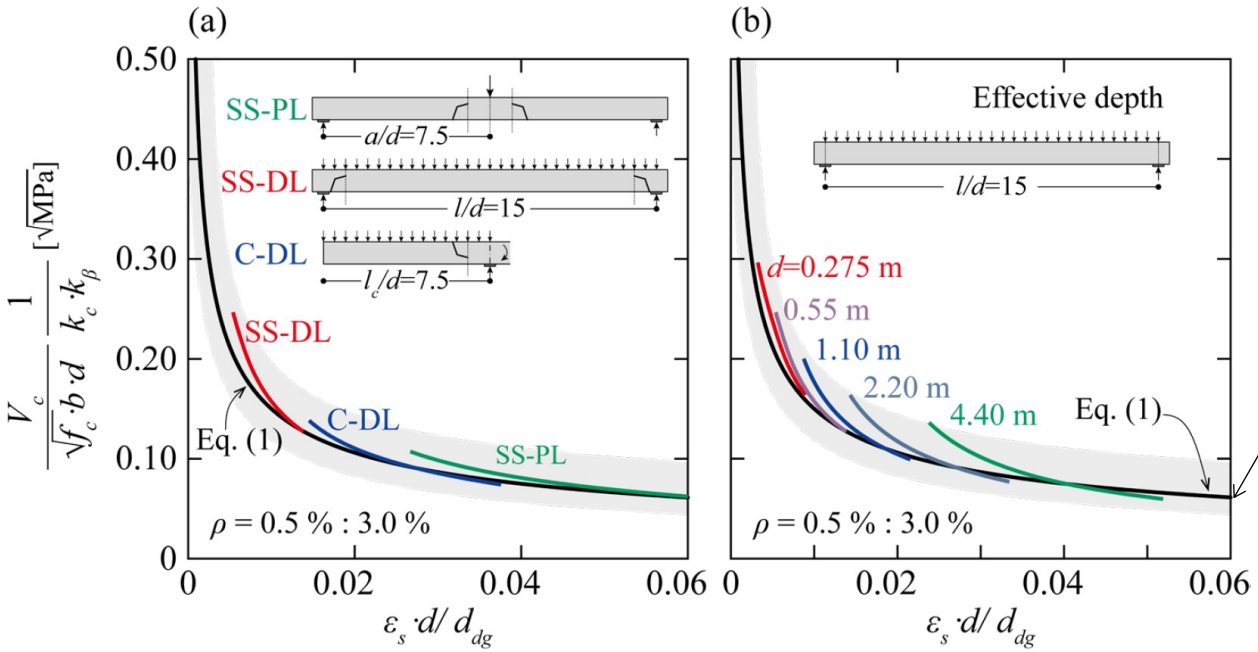


$$V_c = 0.015 \cdot k_c \cdot k_\beta \cdot \left(\frac{d_{dg}}{\varepsilon_s \cdot d}\right)^{\frac{1}{2}} \cdot \sqrt{f_c} \cdot b \cdot d$$

Adapted from Cavagnis et al. 2018

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Bases of the Critical Shear Crack Theory (instance of shear in members without shear reinf.)



Adapted from Cavagnis et al. 2018
(see also Muttoni and Simões, 2023)

$$V_c = 0.015 \cdot k_c \cdot k_\beta \cdot \left(\frac{d_{dg}}{\varepsilon_s \cdot d}\right)^{\frac{1}{2}} \cdot \sqrt{f_c} \cdot b \cdot d \quad (1)$$

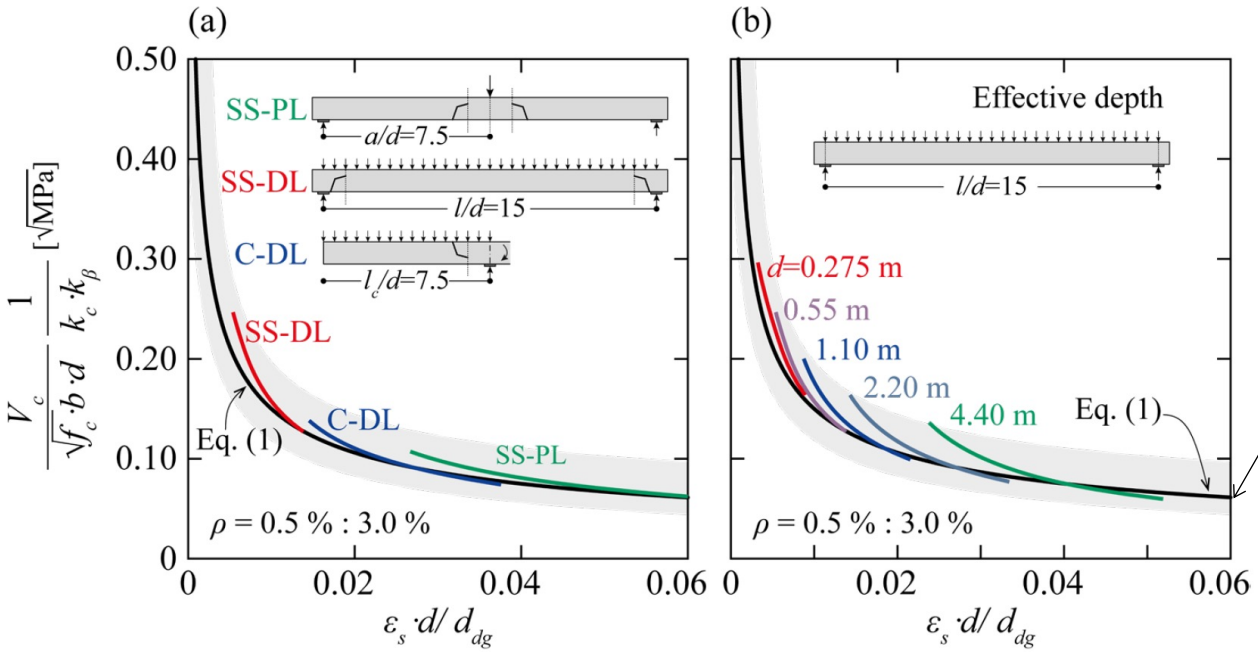
$$\varepsilon_s = \frac{V_E \cdot a_{cs}}{z \cdot \underbrace{\rho \cdot b \cdot d}_{A_s} \cdot E_s} \qquad a_{cs} = \frac{M_{Ed}}{V_{Ed}}$$

$$\frac{V_R}{b \cdot d} = 0.75 \cdot \left(k_c \cdot k_\beta\right)^{2/3} \cdot \left(100 \cdot \rho \cdot f_c \frac{d_{dg}}{a_{cs}}\right)^{\frac{1}{3}}$$

$$\frac{V_R}{b \cdot d} = 0.75 \cdot \left(100 \cdot \rho \cdot f_c \frac{d_{dg}}{\sqrt{d \cdot a_{cs}}}\right)^{\frac{1}{3}}$$

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$$V_c = 0.015 \cdot k_c \cdot k_\beta \cdot \left(\frac{d_{dg}}{\varepsilon_s \cdot d}\right)^{\frac{1}{2}} \cdot \sqrt{f_c} \cdot b \cdot d \quad (1)$$

$$\varepsilon_s = \frac{f_y}{E_s}$$

$$\tau_{Rc,\min} = 0{,}015 k_c k_\beta \sqrt{\frac{f_c E_s}{f_y} \frac{d_{dg}}{d}} \frac{d}{z} \longrightarrow \tau_{\mathrm{Rdc,min}} = \frac{11}{\gamma_V} \cdot \sqrt{\frac{f_{ck}}{f_{yd}} \cdot \frac{d_{dg}}{d}}$$

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Bases of the Critical Shear Crack Theory (instance of shear in members without shear reinf.)

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REVIEW PAPER

Shear and punching shear according to the Critical Shear Crack Theory: background, recent developments and integration in codes

Corte e punçoamento de elementos de betão armado de acordo com a Teoria da Fissura de Corte Crítica: enquadramento histórico, desenvolvimentos recentes e integração em normas

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Muttoni and Simões, 2023

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Refined verification of the shear stress resistance, two levels of refinement

$$\frac{V_R}{b \cdot d} = 0.75 \cdot \left(100 \cdot \rho \cdot f_c \frac{d_{dg}}{\sqrt{d \cdot a_{cs}}}\right)^{\frac{1}{3}}$$

Formula derived analytically from Critical Shear Crack Theory

$$\tau_{\mathrm{Rd,c}} = \frac{0{,}66}{\gamma_{\mathrm{V}}} \cdot \left(100 \rho_{\mathrm{l}} \cdot f_{\mathrm{ck}} \cdot \frac{d_{\mathrm{dg}}}{d}\right)^{\frac{1}{3}} \geq \tau_{\mathrm{Rdc,min}}$$

Formula in prEN 1992-1-1:2023 with some simplifications ($\sqrt{d \cdot a_{cs}}$ replaced by d) and after reliability analysis to calibrate γ_{V}

Possibility to increase the resistance by accounting for the shear span

(3) The value of d in Formula (8.27) may be replaced by the mechanical shear span a_{v}, for members with an effective shear span a_{cs} shorter than 4 d:

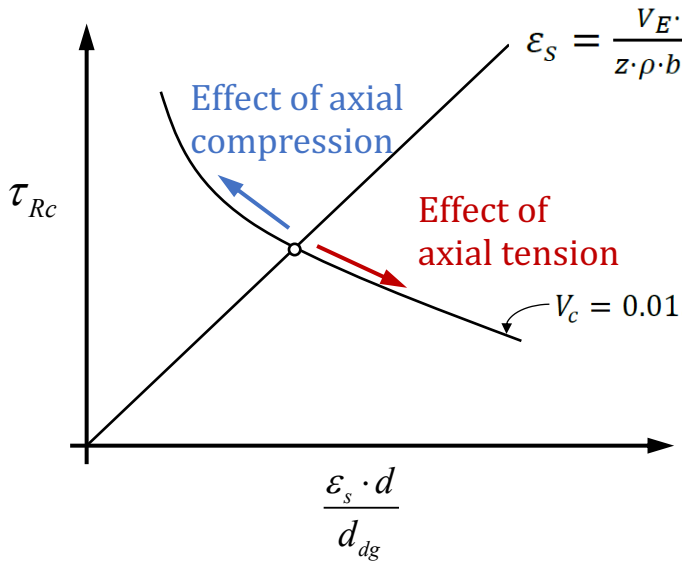
$$a_{\mathrm{v}} = \sqrt{\frac{a_{\mathrm{cs}}}{4} \cdot d} \qquad (8.29)$$

Where a_{cs} is the effective shear span with respect to the control section. For reinforced concrete members it may be calculated as a function of the internal forces at control section:

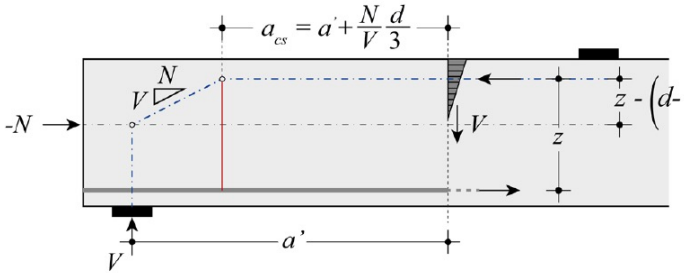
$$a_{\mathrm{cs}} = \left|\frac{M_{\mathrm{Ed}}}{V_{\mathrm{Ed}}}\right| \geq d \qquad (8.30)$$

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Effect of the axial force on the shear resistance of members without shear reinforcement



$$V_c = 0.015 \cdot k_c \cdot k_\beta \cdot \left(\frac{d_{dg}}{\varepsilon_s \cdot d}\right)^{\frac{1}{2}} \cdot \sqrt{f_c} \cdot b \cdot d$$



Force in the reinforcement

$$V\frac{a'}{z} + N\frac{z-(d-h/2)}{z} = V\frac{a_{cs}}{z}$$

Muttoni and Simões, 2023

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Effect of the axial force on the shear resistance of members without shear reinforcement

(4) In presence of axial forces N_{Ed}, acting at the control section, the value of d in Formula (8.27) or a_v in Formula (8.29) should be multiplied by coefficient k_{vp} according to Formula (8.31):

$$k_{vp} = 1 + \frac{N_{Ed}}{|V_{Ed}|}\frac{d}{3 \cdot a_{cs}} \geq 0{,}1 \qquad (8.31)$$

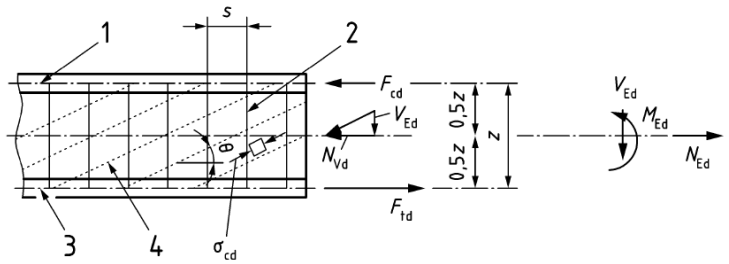
For an alternative approach for members with compressive axial forces, see presentation by prof. Pedro Miguel

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Shear in members with shear reinforcement

8.2.3 Members with shear reinforcement

(1) The design of members with shear reinforcement should be based on a compression field (Figure 8.9). Limiting values for the angle θ of the inclined compression field in the web are given in (4). Provisions for inclined shear reinforcement are given in (13).

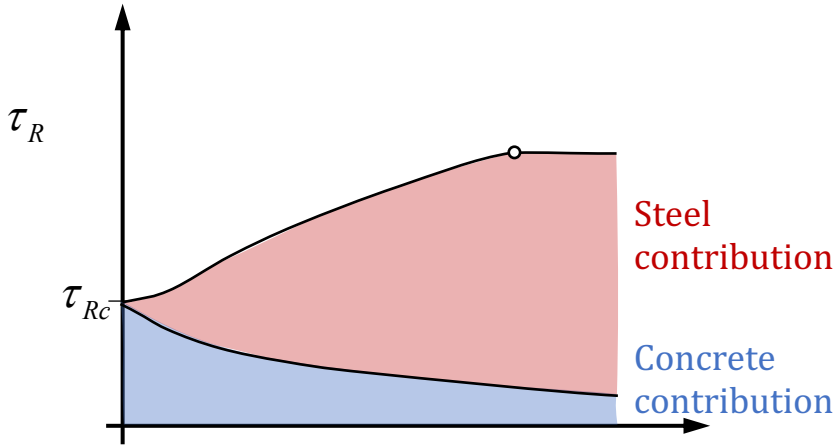


- Model in current EN 1992-1-1:2004 confirmed
- Two levels of refinement
- More consistent model for concentrated loads applied near to supports

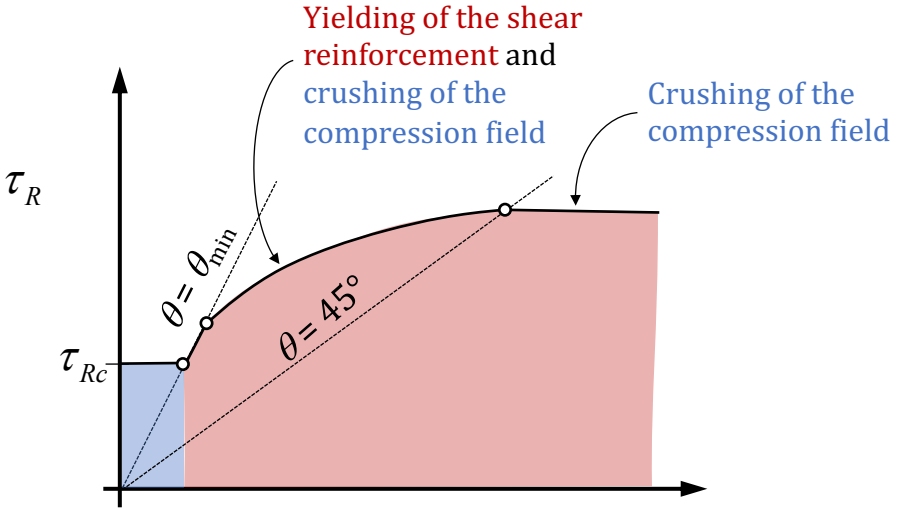
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Shear in members with shear reinforcement, potential models

Models accounting for concrete and steel contributions (for instance Modified Compression Field Theory, Collins-Vecchio-Bentz, Toronto)

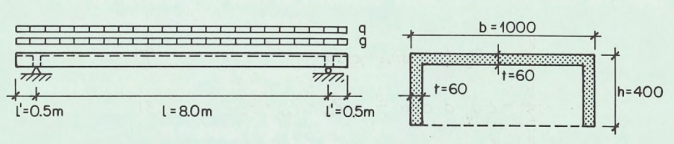


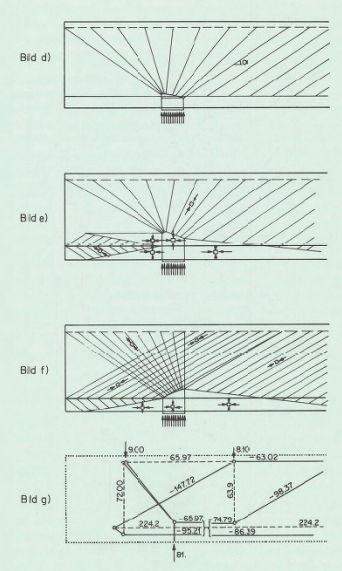
Models based on limit analysis (B. Thürlimann in Zürich and M.P. Nielsen in Copenhagen, adopted in *fib* MC1978 and EN 1992-1-1:2004)

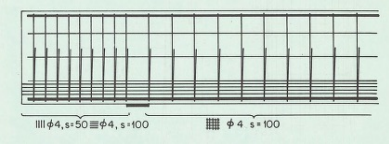


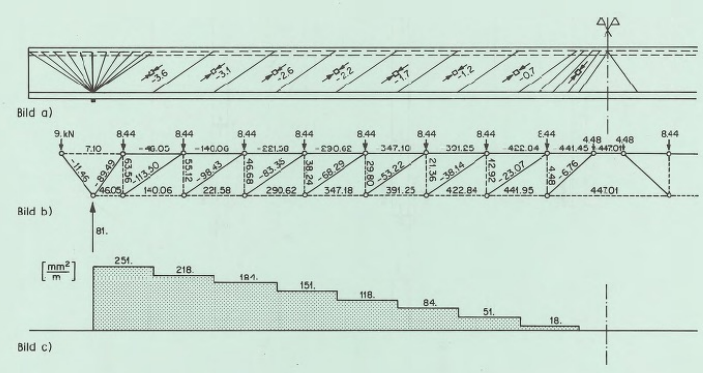
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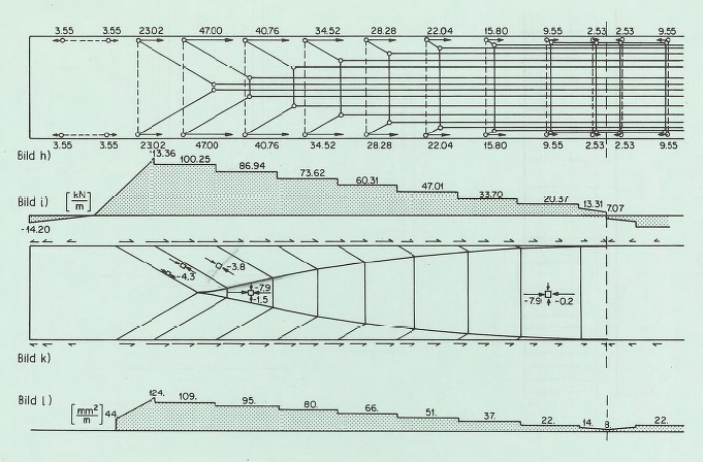
Models based on limit analysis: fully consistent with strut and tie models and stress fields







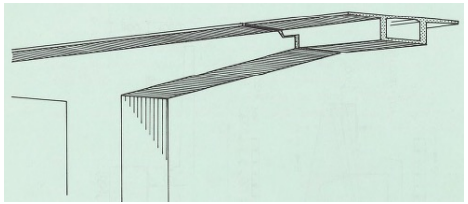


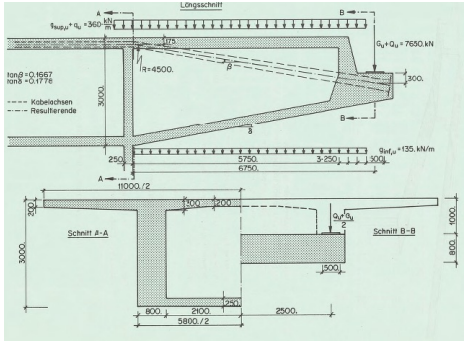


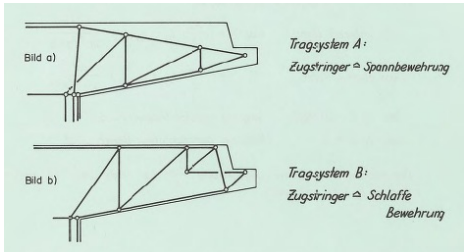
Muttoni 1984

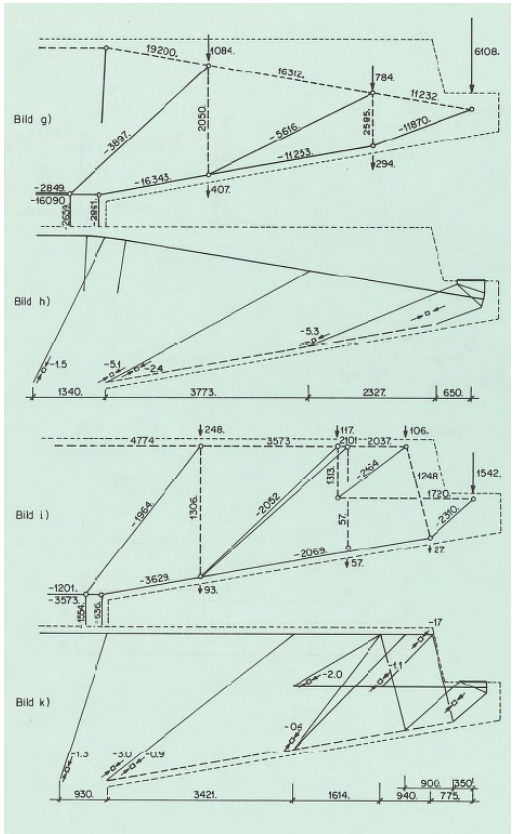
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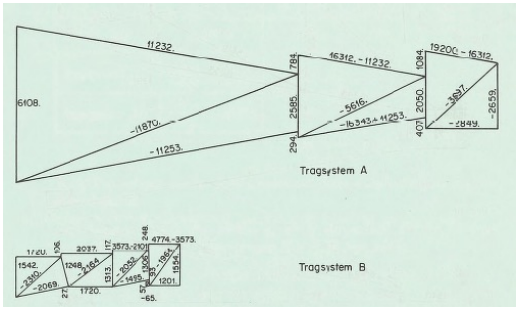
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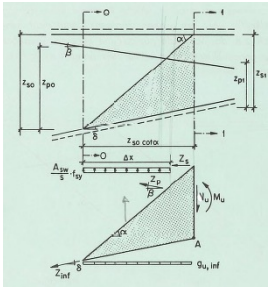


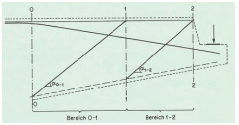












Muttoni 1984

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Shear in members with shear reinforcement, two levels of refinement

- Design of shear reinforcement ……

$$\tau_{Rd,sy} = \rho_w \cdot f_{ywd} \cdot \cot\theta \qquad \rho_w = \frac{A_{sw}}{b_w \cdot s}$$

and verification of the concrete stress in the compression field

$$\sigma_{cd} = \tau_{Ed}(\cot\theta + \tan\theta) \leq \nu \cdot f_{cd}$$

- 1st level of refinement

$$\nu = 0{,}5$$

(4) The inclination of the compression field in the web carrying shear may be selected within the following range:

$$1 \leq \cot\theta \leq \cot\theta_{min} \qquad (8.41)$$

where the cotangent of the minimal inclination of the compression field θ_{min} should be for shear reinforcement of ductility class B or C:

— $\cot\theta_{min} = 2{,}5$ for ordinary reinforced members without axial force;

— $\cot\theta_{min} = 3{,}0$ for members subjected to significant axial compressive force (average axial compressive stress $\geq |3\ MPa|$) and provided that the depth of the compression chord x determined from a sectional analysis according to 8.1.1 and 8.1.2 is less than $0{,}25d$. Interpolated values between 2,5 and 3,0 may be adopted for intermediate cases. For very high compressive forces ($x > 0{,}25d$), (11) can apply;

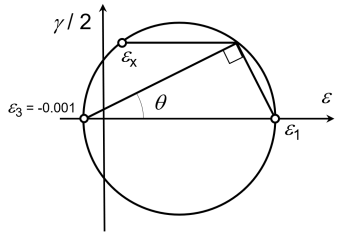
— $\cot\theta_{min} = 2{,}5 - 0{,}1 \cdot N_{Ed}/|V_{Ed}| \geq 1{,}0$ for members subjected to axial tension.

For shear reinforcement of ductility class A, $\cot\theta_{min}$ shall be reduced by 20 %.

- 2nd level of refinement

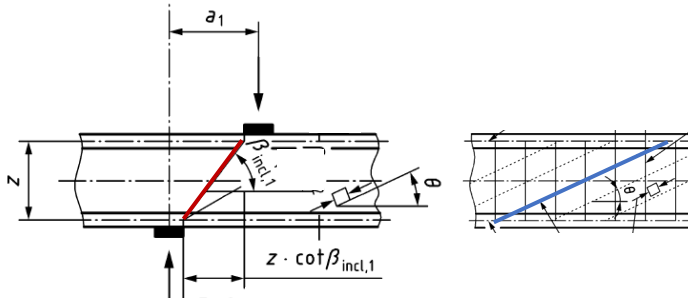
Angles of the compression field inclination lower than θ_{min} or values of coefficient ν higher than 0,5 may be adopted provided that ν is calculated on the basis of the state of strains according to:

$$\nu = \frac{1}{1{,}0 + 110 \cdot (\varepsilon_x + (\varepsilon_x + 0{,}001) \cdot \cot^2\theta)} \leq 1{,}0$$



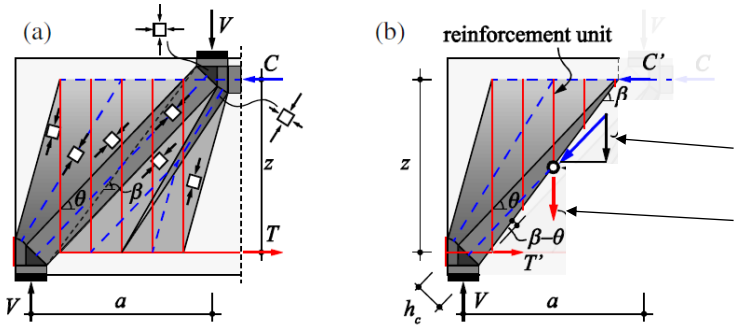
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Shear in members with shear reinforcement, concentrated loads applied near to supports



The general model is not correct (overly conservative) in case $\cot\beta < \cot\theta$

Exact solution according to the stress field method



$$\tau_{Rd} = \nu \cdot f_{cd} \frac{\cot\theta - \cot\beta_{incl}}{1 + \cot^2\theta} + \rho_w \cdot f_{ywd} \cdot \cot\beta_{incl} \leq \nu \cdot f_{cd} \frac{\cot\theta}{1 + \cot^2\theta}$$

Direct strut (concrete contribution)

Shear reinforcement contribution

Pejatovic et al. 2022

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Punching

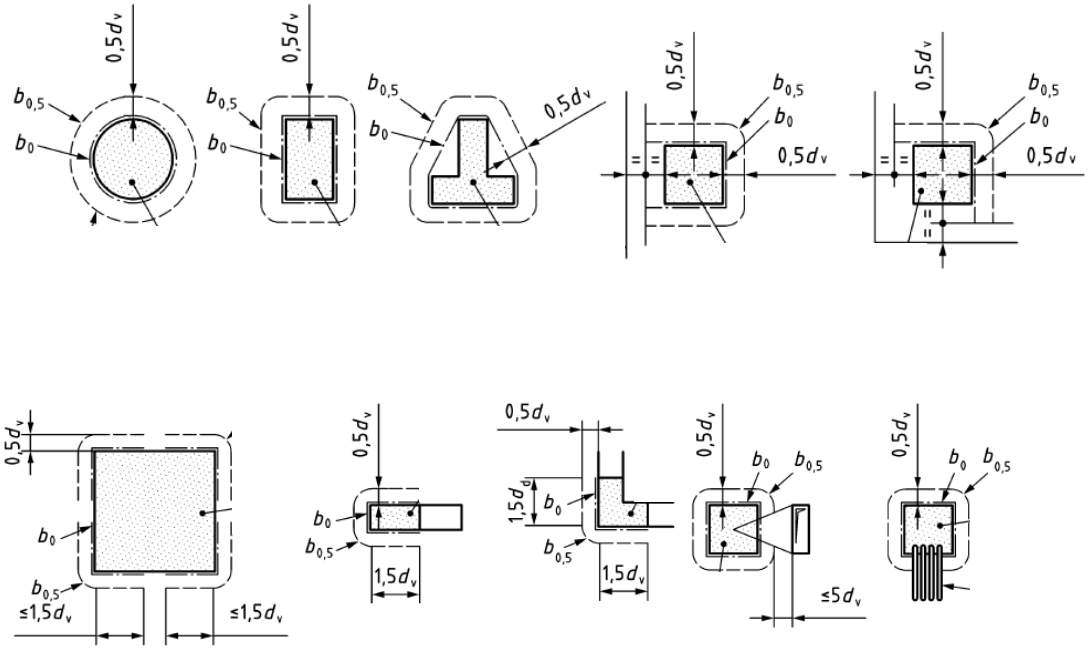
$$\tau_{\mathrm{Rd,c}} = \frac{0{,}6}{\gamma_{\mathrm{V}}} \cdot k_{\mathrm{pb}} \left(100\, \rho_{\mathrm{l}} \cdot f_{ck} \cdot \frac{d_{\mathrm{dg}}}{d_{\mathrm{v}}} \right)^{\frac{1}{3}} \leq \frac{0{,}5}{\gamma_{\mathrm{V}}} \cdot \sqrt{f_{\mathrm{ck}}}$$

- Formula looks similar as in current EN 1992-1-1:2004
- What is new?

- Formula derived analytically from the general expression of the Critical Shear Crack Theory (formula in current EN 1992-1-1:2004 was fully empirical)
- Several levels of refinement
- New definition of control perimeter and size-effect law
- Punching of slabs with shear reinforcement (formulae also derived from CSCT)
- Refined verification according to CSCT combined with non-linear analysis in Annex I (assessment of existing structures)

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Punching, new definition of control perimeter

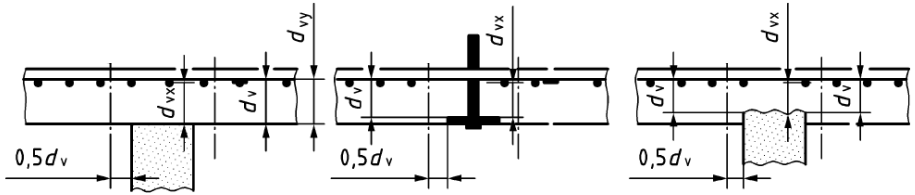


a) around large supporting areas b) around wall end c) around wall corner d) near opening e) with inserts

- Control perimeter $b_{0,5}$ located at a distance of $0{,}5d_v$ from the column edge (more consistent with the general model of CSCT and allows for the same verification of flat slabs and foundation slabs, no iteration require for the latter)
- Perimeter b_0 at the column edge also used in some formulae
- Reduction for large columns
- Definition for wall ends and wall corners

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Punching, definition of shear resisting effective depth d_v



a) direct support

b) hanging support with penetration

c) penetration by direct support

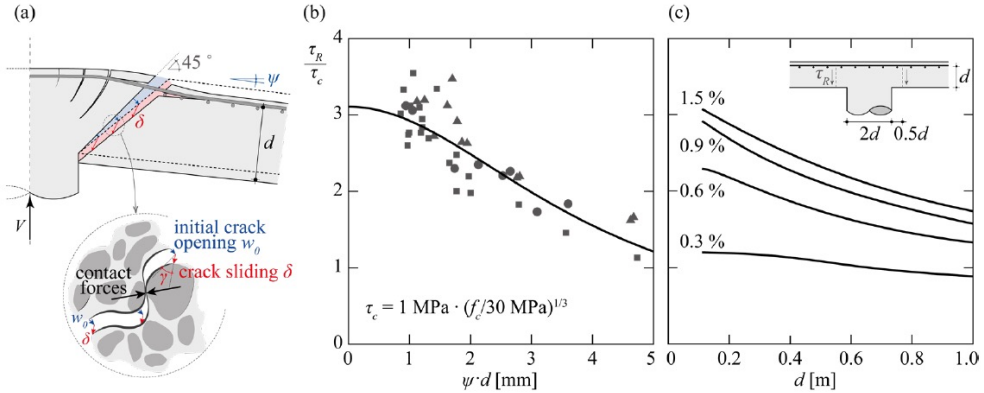
- Reduction of shear resisting effective depth in case of support with penetration (typical case of anchored walls or steel columns with embedded steel plates)
- Reduction when the column penetrates into the slab by more than $d/20$



Muttoni et al., 2005

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Punching, bases of the Critical Shear Crack Theory



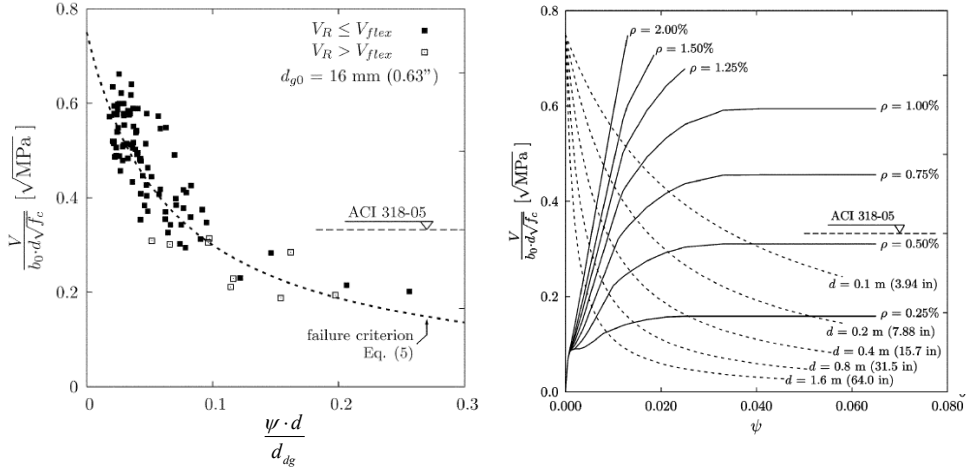
$$V_R = \frac{b_{0,5} \cdot d \cdot f_c^{1/3}}{1 + (\psi \cdot d / 4\text{mm})^2}$$

- Main assumption of the original model: opening of the critical shear crack proportional to $\psi \cdot d$ (confirmed by detailed measurements in the following)
- This allows to account for the size and strain effect in a consistent manner
- The rotation ψ can be calculated on the basis of the flexural deformations of the slab → the punching shear resistance can be calculated
- Simplified version accounting for the size effect implemented in Swiss standard SIA 162 of 1993

adapted from Muttoni and Schwartz, 1991
(see also Muttoni and Simões, 2023)

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Punching, bases of the Critical Shear Crack Theory



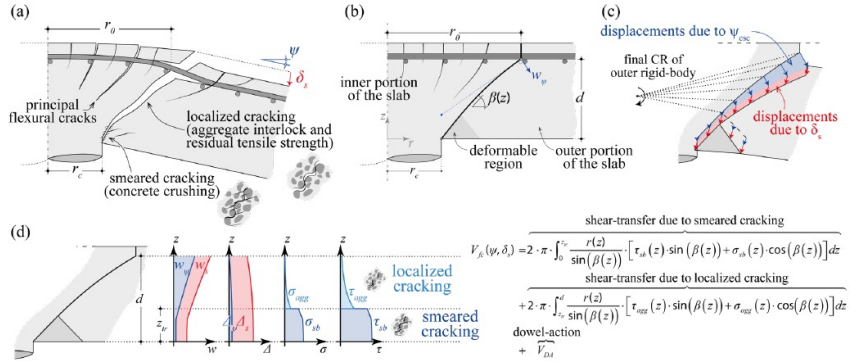
$$V_R = \frac{0.75 \cdot b_{0,5} \cdot d \cdot f_c^{1/2}}{1 + 15 \cdot \psi \cdot d / d_{dg}}$$

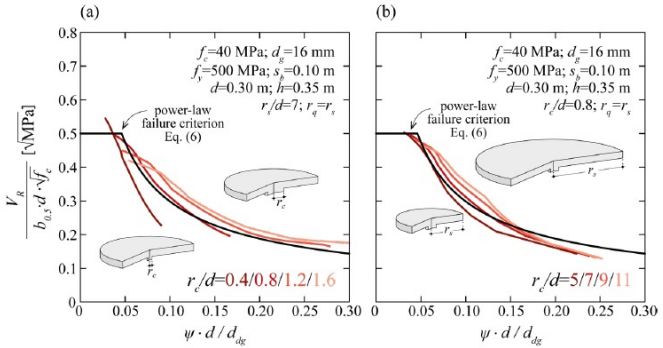
- Refined of the failure criterion accounting for the aggregate size
- Simplified version implemented in Swiss standard of 2003 and in the *fib* MC 2010

Muttoni 2003 and Muttoni 2008

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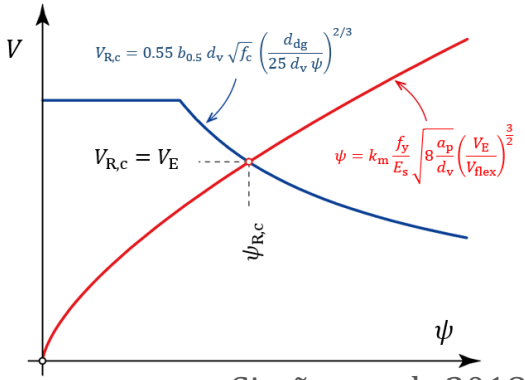
Punching, bases of the Critical Shear Crack Theory





$$\frac{V_{R,c}}{b_{0.5}\cdot d_v\cdot\sqrt{f_c}} = 0.55\cdot\left(\frac{d_{dg}}{25\cdot\psi\cdot d}\right)^{\frac{2}{3}} \le k_F$$

- Refined model by Simões et al. 2018 has shown that the failure criterion can be expressed by a power law
- A closed-form solution can be derived analytically in a similar manner as for shear (Muttoni et al. 2018) leading to the new formula for the 2nd generation of EC2



Simões et al., 2018

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Punching shear stress τ_{Ed} according to FprEN 1992-1-1:2023

$$\tau_{Ed} = \beta_e \frac{V_{Ed}}{b_{0,5} \cdot d_v}$$

- Control perimeter located at $0.5 \cdot d_v$ from the column edge
- Two levels of refinement for coefficient β_e which accounts for a non-constant distribution of shear forces along the control perimeter

Table 8.3 — Coefficients β_e accounting for concentrations of the shear forces

Support	Approximated	Refined[a]	
internal columns	$\beta_e = 1{,}15$	$\beta_e = 1 + 1{,}1\frac{e_b}{b_b} \geq 1{,}05$	where $e_b = \sqrt{e_{b,x}^2 + e_{b,y}^2}$
edge columns	$\beta_e = 1{,}4$		where $e_b = 0{,}5\lvert e_{b,x}\rvert + \lvert e_{b,y}\rvert$
corner columns	$\beta_e = 1{,}5$		where $e_b = 0{,}27(\lvert e_{b,x}\rvert + \lvert e_{b,y}\rvert)$
ends of walls	$\beta_e = 1{,}4$		
corners of walls	$\beta_e = 1{,}2$		

D. Abu-Salma et al., 2023

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Punching shear resistance τ_{Rd} according to FprEN 1992-1-1:2023

$$\tau_{\mathrm{Rd,c}} = \frac{0{,}6}{\gamma_{\mathrm{V}}} \cdot k_{\mathrm{pb}} \left(100\, \rho_{\mathrm{l}} \cdot f_{ck} \cdot \frac{d_{\mathrm{dg}}}{d_{\mathrm{v}}} \right)^{\frac{1}{3}} \leq \frac{0{,}5}{\gamma_{\mathrm{V}}} \cdot \sqrt{f_{\mathrm{ck}}}$$

$$1 \leq k_{\mathrm{pb}} = 3{,}6 \sqrt{1 - \frac{b_0}{b_{0,5}}} \leq 2{,}5$$

Possibility to increase the resistance by accounting for the shear span

- Formula similar to the one for the shear resistance, but with an additional term to account for the effect of the column size
- Two levels of refinement for the size and slender effect

(2) For distances between the centre of the support area and the point of contraflexure in the considered load combination a_{p} smaller than $8d_{\mathrm{v}}$, the value of d_{v} in Formula (8.94) may be replaced by:

$$a_{\mathrm{pd}} = \sqrt{\frac{a_{\mathrm{p}}}{8} \cdot d_{\mathrm{v}}} \qquad (8.97)$$

where

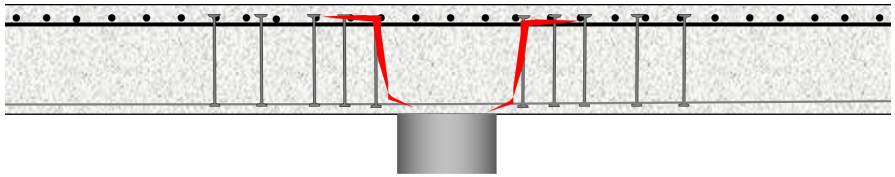
$$a_{\mathrm{p}} = \sqrt{a_{\mathrm{p,x}} \cdot a_{\mathrm{p,y}}} \geq d_{\mathrm{v}} \qquad (8.98)$$

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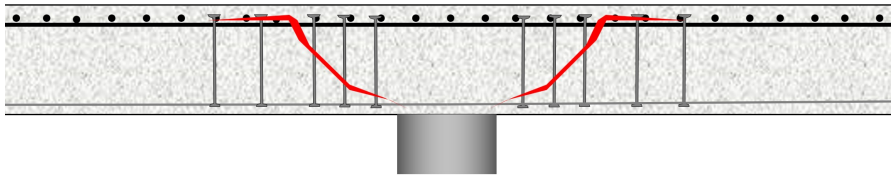
Punching shear of slabs with shear reinforcement

3 potential failure modes → 3 verifications required

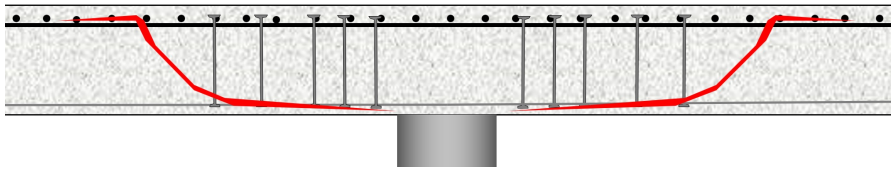
Maximum punching resistance $\tau_{Rd,max}$

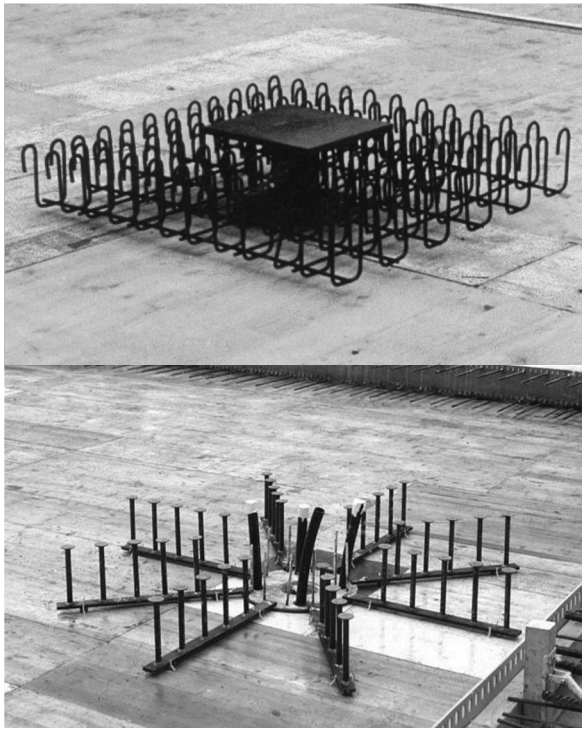


Punching resistance inside the reinforced zone $\tau_{Rd,cs}$



Punching resistance outside the reinforced zone $\tau_{Rd,c}$





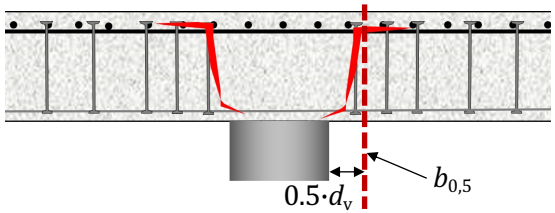
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Maximum punching shear resistance $\tau_{Rd,max}$

$$\tau_{Rd,max} = \eta_{sys} \cdot \tau_{Rd,c}$$

$$\eta_{sys} = 0{,}70 + 0{,}63 \left(\frac{b_0}{d_v}\right)^{1/4} \geq 1{,}0 \text{ for studs}$$

$$\eta_{sys} = 0{,}50 + 0{,}63 \left(\frac{b_0}{d_v}\right)^{1/4} \geq 1{,}0 \quad \text{for links and stirrups}$$



Hernández et al., 2021

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Punching shear resistance $\tau_{Rd,s}$ within the shear reinforced zone

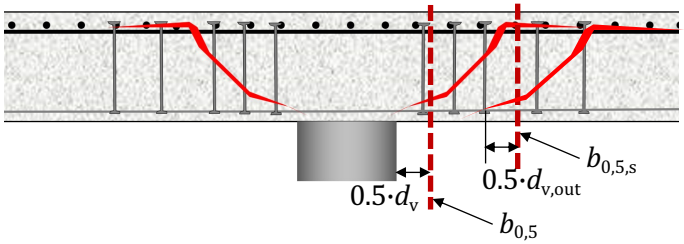
$$\tau_{Rd,cs} = \eta_c \cdot \tau_{Rd,c} + \eta_s \cdot \rho_w \cdot f_{ywd} \geq \rho_w \cdot f_{ywd}$$

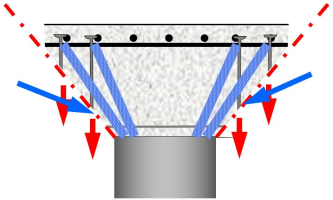
where

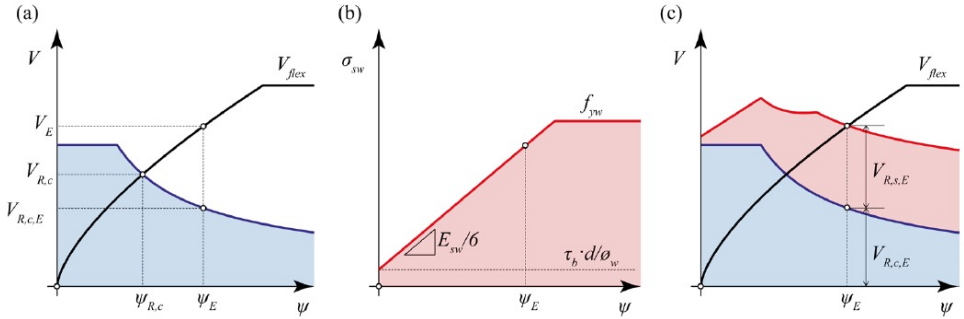
$$\eta_c = \frac{\tau_{Rd,c}}{\tau_{Ed}}$$

$\tau_{Rd,c}$ is the punching shear stress resistance of slabs without shear reinforcement according to Formula (8.94).

$$\eta_s = \frac{d_v}{150\phi_w} + \left(15\frac{d_{dg}}{d_v}\right)^{1/2} \cdot \left(\frac{1}{\eta_c \cdot k_{pb}}\right)^{3/2} \leq 0{,}8$$







Muttoni and Simões, 2023

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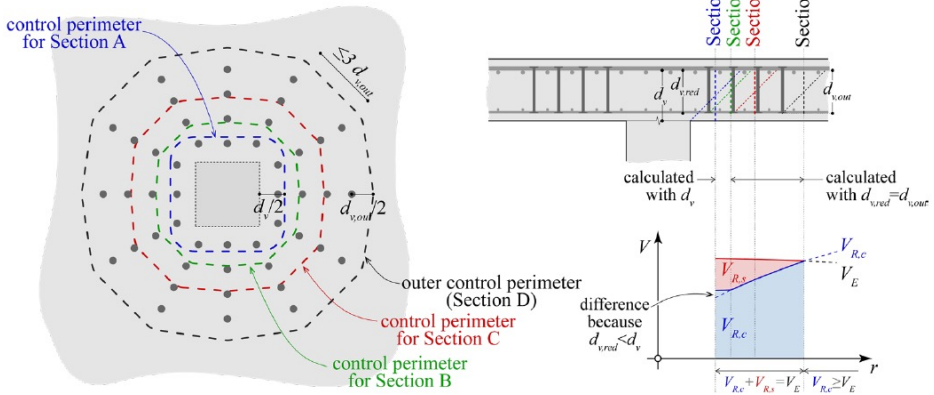
Punching outside the shear reinforced zone

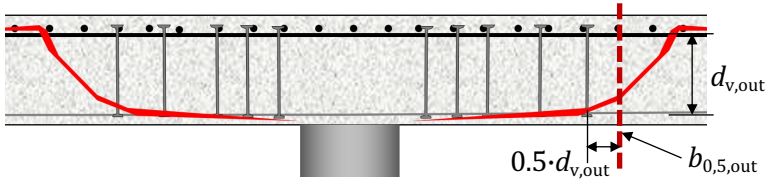
Two levels of refinement:

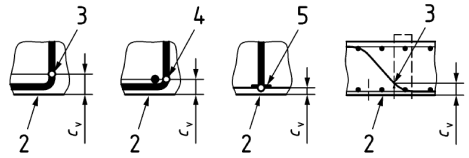
- Direct calculation of the required length of the control perimeter:

$$b_{0,5,\mathrm{out}} = b_{0,5} \cdot \left(\frac{d_\mathrm{v}}{d_{\mathrm{v,out}}} \cdot \frac{1}{\eta_\mathrm{c}} \right)^2$$

- Explicit verification:







2 limit of bend
3 level of axis of reinforcement inside the bend
4 bottom of stud head

Muttoni and Simões, 2023

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References related to the Critical Shear Crack Theory for punching in FprEN 1992-1-1:2023

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Disponible en www.hormigonyacero.com
Hormigón y Acero 2023; 74(299-300):61-77
https://doi.org/10.33586/hya.2022.3091

A Mechanical Approach for the Punching Shear Provisions in the Second Generation of Eurocode 2

El tratamiento del punzonamiento en la segunda generación del Eurocódigo 2 basado en un modelo mecánico

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ABSTRACT

The revision of a code is a long-term project that shall fulfil several aims, comprising the enhancement of the ease-of-use and incorporating updated state-of-the-art. With respect to the revision of Eurocode 2 concerning the punching shear provisions, this task allowed also for the opportunity to enhance the understanding of the code and physical phenomenon by designers. The original EN1992-1-1:2004 punching provisions were adapted from an empirical equation for design based on the regression analyses performed by Zsutty in the 1960s for shear in beams and later reworked in Model Code 1990 for punching shear. These expressions did not show any link to the physical response of a structure, making difficult to designers to clearly understand how to engineer their designs. Instead of continuing with this approach, CEN/TC250/WG1 took the decision in 2016 to ground the punching provisions on a mechanical model that could be explained to engineers, allowing for a transparent understanding of the design equations and phenomena. To that aim, the Critical Shear Crack Theory, already implemented in Model code 2010 at that time, was selected as representative of the state-of-the-art. Following that decision, a large effort has been performed to implement this theory into the Eurocode, keeping its simplicity of use and generality. This paper is aimed at presenting the theoretical grounds of the theory as well as the manner in which it is drafted for the future generation of Eurocode 2.

KEYWORDS: Punching, shear, reinforced concrete, slabs, column bases, mechanical model.

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REVIEW PAPER

Shear and punching shear according to the Critical Shear Crack Theory: background, recent developments and integration in codes

Corte e punçoamento de elementos de betão armado de acordo com a Teoria da Fissura de Corte Crítica: enquadramento histórico, desenvolvimentos recentes e integração em normas

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Abstract: The Critical Shear Crack Theory (CSCT) has been developed since 1985 to assess the shear resistance of members without shear reinforcement and the punching shear resistance of reinforced concrete slabs in a rational manner. The main idea of the CSCT is that the shear resistance is governed by the development of a critical shear crack, its geometry and its kinematics. Recent shear tests with detailed measurements have confirmed that the shear force can be carried through the critical shear crack by a combination of aggregate interlocking, residual tensile strength of concrete, dowel action of the longitudinal reinforcement, inclination of the compression zone and activation of the shear reinforcement crossed by the critical shear crack if present. On the basis of advanced constitute laws, all these contributions can be calculated as a function of the crack geometry and its kinematic. Simplifications of the resulting general formulations have been implemented in several standards including the *fib* Model Code 2010 and, in its recent closed-form format, in the second generation of the European Standard for Concrete Structures. The generality of the models allows accounting for several materials and cases, as for instance the presence of axial forces, fiber reinforced concrete, non-metallic reinforcements and designing strengthening using several techniques. This document presents the historical framework of the development of the theory, followed by a short presentation of its most up-to-date refined models. The derivation of closed-form solutions based on the CSCT and how it leads to expressions in a format similar to the current European Standard for Concrete Structures is also discussed. Eventually, for the case of punching, some recent developments are shown in what refers the capability of the refined mechanical model to capture the relationship between the acting punching load, the rotation and the shear deformation during loading and at failure.

Keywords: shear, punching shear, mechanical model, codes, levels-of-approximation.

Muttoni and Simões, 2023

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Thank you for your attention

Aurelio Muttoni

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