Stress field and strut-and tie models

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Universidad Politécnica de Madrid

EUROCODES

EN 1992

Design
of concrete
structures

ACHE
Asociación Española de
Ingeniería Estructural

2nd generation of Eurocode 2 on concrete structures

Madrid, October 17th, 2023

HORMIGÓN
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1. Introduction

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Stress Fields and Strut-and-Tie Models as a Basic Tool for Design and Verification in Second Generation of Eurocode 2

Campos de tensiones y modelos de bielas y tirantes: herramientas fundamentales para el proyecto y comprobación de estructuras de hormigón en la segunda generación del Eurocódigo 2

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ABSTRACT

Within the frame of the revision of the Eurocode 2 for concrete structures, the section devoted to strut-and-tie design has been updated to enhance its applicability, its consistency with other sections and its ease-of-use. As a result, a number of changes have been introduced. Namely, the use of stress fields and their combination with classical strut-and-tie models has been incorporated. The changes in this section can be seen as an effort to provide a more comprehensive and general tool for designers, that can be transparently applied to any structural member with sufficient reinforcement for crack control. In this paper, the consistency between the strut-and-tie and the stress field methods is clarified as well as the fundamentals of the revision performed in Eurocode 2. The paper also elaborates how the code can be used for advanced analyses, considering in an explicit manner the compatibility of deformations to obtain refined estimates of the structural resistance..

KEYWORDS: Strut-and-tie, stress fields, limit analysis, shear, discontinuity regions, shell design.

RESUMEN

En el marco de la revisión del Eurocódigo 2 para estructuras de hormigón, se ha actualizado el capítulo dedicado al diseño mediante modelos de bielas y tirantes, mejorando su coherencia con otras secciones así como su facilidad de uso. Para ello, se han introducido una serie de cambios, como la consideración del método de campos de tensiones y su combinación con los modelos clásicos de bielas y tirantes. Estos cambios pueden considerarse como un esfuerzo para proporcionar una herramienta más completa y general. En este artículo, se clarifica la coherencia entre los métodos de bielas y tirantes y los campos de tensiones, así como los principios de la revisión efectuada para el Eurocódigo 2. También se explica cómo puede utilizarse dicha norma para realizar análisis avanzados, considerando de manera explícita la compatibilidad de deformaciones con el objetivo de obtener estimaciones precisas de la resistencia.

PALABRAS CLAVE: Bielas y tirantes, campos de tensiones, análisis límite, cortante, regiones de discontinuidad, diseño de lajas.

1. INTRODUCTION

Reinforced concrete as a structural material was introduced at the end of the 19th century through a number of patents [1-3].

Almost from the beginning, the engineers realised that traditional design methods rooted in linear elastic theory could not adequately be used to explore the full potential of the new composite material. A new approach was therefore needed and within

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respect (at nodes A and C) the minimum angle $\theta_{cs} = 20°$ recommended by the FprEN1992-1-1:2022.

Grounded on these observations, some refinements can be introduced in the strut-and-tie model, Figure 9e, by providing spreading of the struts. Such spreading allows fulfilling the requirements in terms of minimum angles between struts and ties, and needs the arrangement of additional reinforcement in the form of horizontal and vertical bars (stirrups or pins, see Figure 9f). With respect to the region at the right of the bent of the bar, the fan region (with a steeper angle of the resulting strut) and the constant-angle compression field region (with a flatter angle of the resulting strut) can be designed following the standard procedure for shear in members with transverse reinforcement.

Finally, detailed checks can be performed on the basis of stress fields at the critical regions (nodal regions A and B), ensuring that sufficient space is available for development of the struts and nodal regions. To that aim, a constant and safe value of the efficiency factor is adopted ($\nu = 0.55$ accounting for the angles of the struts and ties), allowing to analyse all nodes under plane-stress hydrostatic conditions, see Figure 9f. The results show that this aspect is not critical. Also, detailing of the reinforcement can be consistently established, in terms of type of reinforcement and anchorage lengths.

For a final optimisation, or in case the performance of the detailed needed to be assessed, a refined EPSF analysis could also be performed. The results are shown in Figure 10 for two cases. The first considers only inclined reinforcement and stirrups at the right of the bend (Figures 10a-d), corresponding to the reinforcement layout of Figure 9d. The latter considers also an additional horizontal and vertical reinforcement in the discontinuity region (Figures 10e-h), corresponding to the reinforcement layout of Figure 9f. In all cases, the load was applied by means of a stiff plate, distributing it into the concrete surface.

When only inclined reinforcement is provided in the discontinuity region (Figures 10a-d), a similar response to that of Figure 9d results, with an inclined compression field devel-

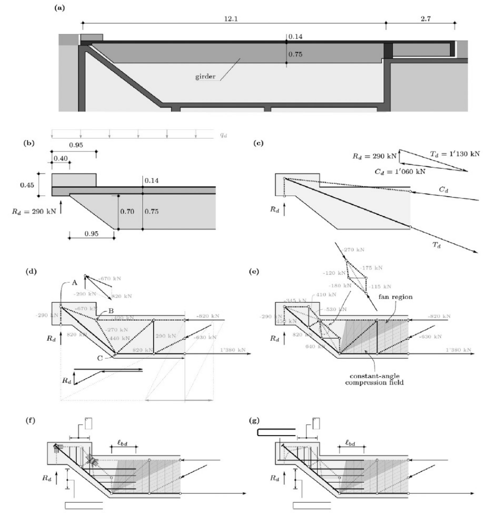

Figure 9. Example of application: (a) element investigated; (b) critical detail; (c) analysis based on thrust line; (d) strut-and-tie model (with lines for analysis with graphic statics shown in light grey); (e) refined strut-and-tie model; (f) stress field verifications and reinforcement layout; and (g) enhanced detailing.

1. Introduction

EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

EN 1992-1-1

December 2004

ICS 91.010.30; 91.080.40

Supersedes ENV 1992-1-1:1991, ENV 1992-1-3:1994, ENV 1992-1-4:1994, ENV 1992-1-5:1994, ENV 1992-1-6:1994, ENV 1992-3:1998

English version

Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings

Eurocode 2: Calcul des structures en béton - Partie 1-1 : Règles générales et règles pour les bâtiments

Eurocode 2: Bemessung und konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau

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Management Centre: rue de Stassart, 36 B-1050 Brussels

Ref. No. EN 1992-1-1:2004: E

(5) Where proprietary products are used as shear reinforcement, $V_{Rd,cs}$ should be determined by testing in accordance with the relevant European Technical Approval. See also 9.4.3.

6.5 Design with strut and tie models

6.5.1 General

(1)P Where a non-linear strain distribution exists (e.g. supports, near concentrated loads or plain stress) strut-and-tie models may be used (see also 5.6.4).

6.5.2 Struts

(1) The design strength for a concrete strut in a region with transverse compressive stress or no transverse stress may be calculated from Expression (6.55) (see Figure 6.23).

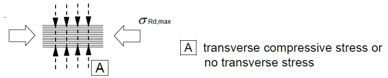


Figure 6.23: Design strength of concrete struts without transverse tension

$$\sigma_{Rd,max} = f_{cd} \quad (6.55)$$

It may be appropriate to assume a higher design strength in regions where multi-axial compression exists.

(2) The design strength for concrete struts should be reduced in cracked compression zones and, unless a more rigorous approach is used, may be calculated from Expression (6.56) (see Figure 6.24).

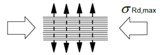


Figure 6.24: Design strength of concrete struts with transverse tension

$$\sigma_{Rd,max} = 0,6\,\nu' f_{cd} \quad (6.56)$$

Note: The value of ν' for use in a Country may be found in its National Annex. The recommended value is given by equation (6.57N).

$$\nu' = 1 - f_{ck}/250 \quad (6.57N)$$

(3) For struts between directly loaded areas, such as corbels or short deep beams, alternative calculation methods are given in 6.2.2 and 6.2.3.

6.5.3 Ties

(1) The design strength of transverse ties and reinforcement should be limited in accordance with 3.2 and 3.3.

(2) Reinforcement should be adequately anchored in the nodes.

EN 1992-1-1 (2004)

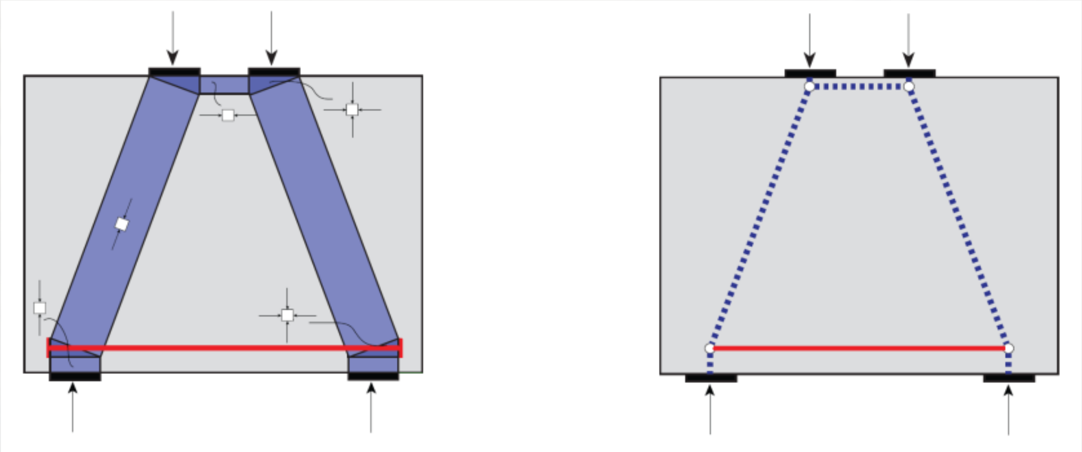
- Strut-and-tie oriented (D-regions)
- 4 pages
- Rules on strut, ties and nodes

1. Introduction

Reasons for change [BD FprEN 1992-1-1:2023]

EN1992-1-1:2004 was inspired in Schlaich's approach to D-regions (1980s). Since then, many advances have been consolidated:

- analogies of strut-and-tie models with the stress field method



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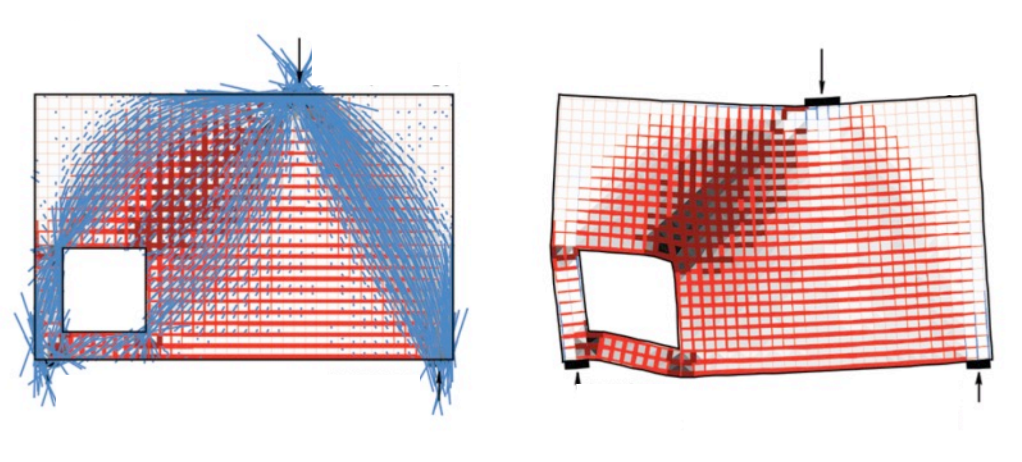
- analogies of strut-and-tie models with the stress field method
- explicit consideration of the strain state in the calculation of efficiency factors



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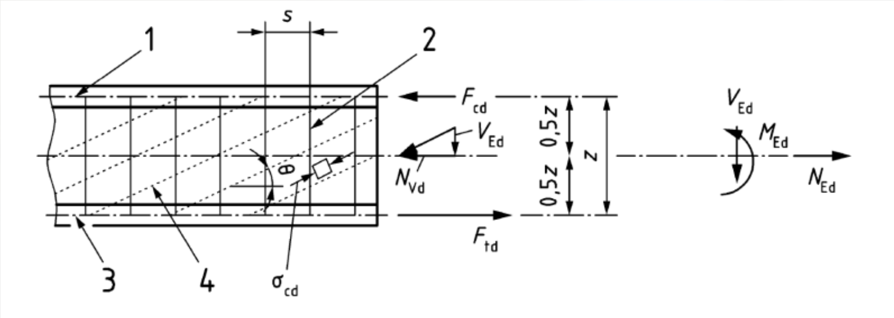
- analogies of strut-and-tie models with the stress field method
- explicit consideration of the strain state in the calculation of efficiency factors
- integration of strut-and-tie models and stress fields within automated design procedures and consistency with design of B-regions



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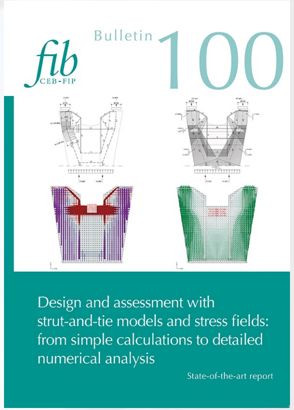
Revision of EN1992-1-1:2004 was based on updated

state-of-the-art presented in Bulletin 100 of fib



1. Introduction

EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

FINAL DRAFT
FprEN 1992-1-1

April 2023

ICS 91.010.30; 91.080.40 — Will supersede EN 1992-1-1:2004

English Version

Eurocode 2 - Design of concrete structures - Part 1-1: General rules and rules for buildings, bridges and civil engineering structures

Eurocode 2 - Calcul des structures en béton - Partie 1-1: Règles générales et règles pour les bâtiments, les ponts et les ouvrages de génie civil

Eurocode 2 - Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1 1: Allgemeine Regeln und Regeln für Hochbauten, Brücken und Ingenieurbauwerke

This draft European Standard is submitted to CEN members for formal vote. It has been drawn up by the Technical Committee CEN/TC 250.

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Recipients of this draft are invited to submit, with their comments, notification of any relevant patent rights of which they are aware and to provide supporting documentation.

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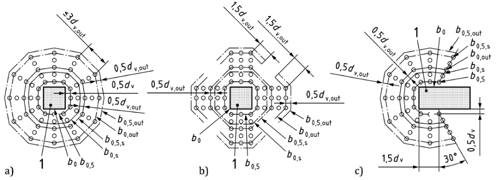
CEN-CENELEC Management Centre: Rue de la Science 23, B-1040 Brussels

Ref. No. FprEN 1992-1-1:2023 E



a) control perimeters for radial arrangements ($b_{0,5,s}$ and $b_{0,s}$ refer to the instance of the 3rd perimeter of shear reinforcement)

b) control perimeters for cruciform arrangements

c) control perimeters for wall ends, wall corners, large and elongated columns (instance of wall end with radial arrangement of shear reinforcement)

Key

1 supporting area

Figure 8.24 — Definition of control perimeters for punching reinforcement

8.5 Design with strut-and-tie models and stress fields

8.5.1 General

(1) Strut-and-tie models or stress fields should be used for design and verification of discontinuity regions (i.e. regions where the strain state distribution is not linear such as near concentrated loads or geometric discontinuities) in absence of specific provisions elsewhere in this code or of alternative refined analyses.

NOTE All provisions of 8.2.3 to 8.2.5 and 8.3 and Annex G are consistent with the rules given in 8.5. Linear members with shear reinforcement and without discontinuities can thus also be designed for bending, shear and torsion with the provisions of 8.5. Unless stated otherwise, the provisions in 8.5 apply to cases that can be modelled under plane stress conditions. Alternative and more refined verification methods for struts, ties and nodes can be adopted for special cases (e.g. where triaxial stress conditions apply).

(2) The provisions in 8.5 are applicable without a verification of the deformation capacity provided that the following conditions are fulfilled:

— the member contains a minimum reinforcement according to Clause 12 or the development of uncontrolled cracks is avoided by other means;

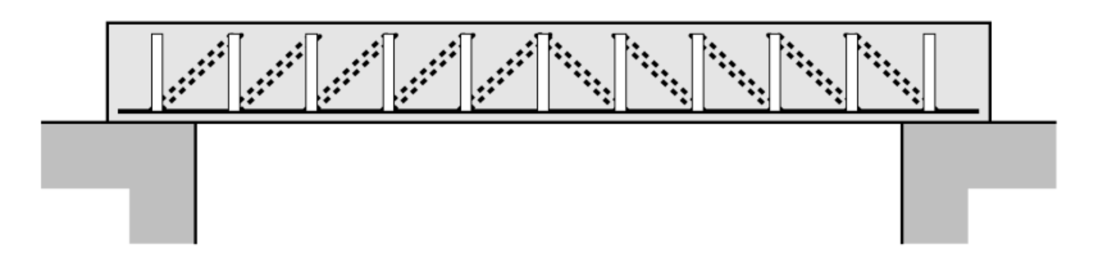
— the reinforcement complies with the details according to Clauses 11 and 12;

— the reinforcing steel is either ductility Class B or C.

FprEN 1992-1-1 (2023)

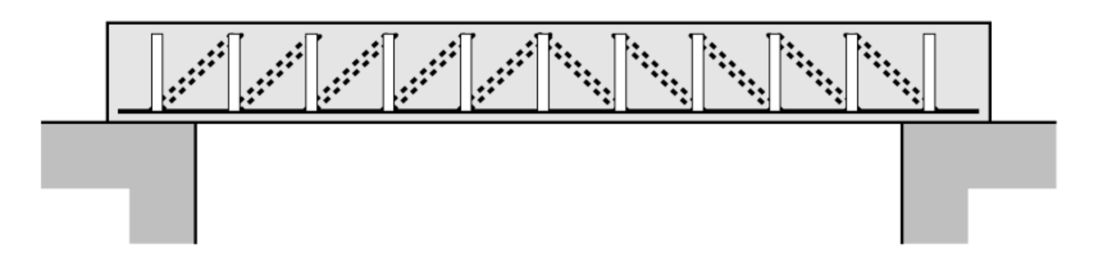
- Strut-and-tie and stress fields oriented
- 9 pages
- Possibility to develop strut-and-tie models and/or stress fields consistently
- Consistent rules with shear design
- Methods to develop equilibrium based strut-and-tie models, rigid-plastic stress fields and more advanced (compatible) stress fields

2. Strut-and-tie and stress fields models

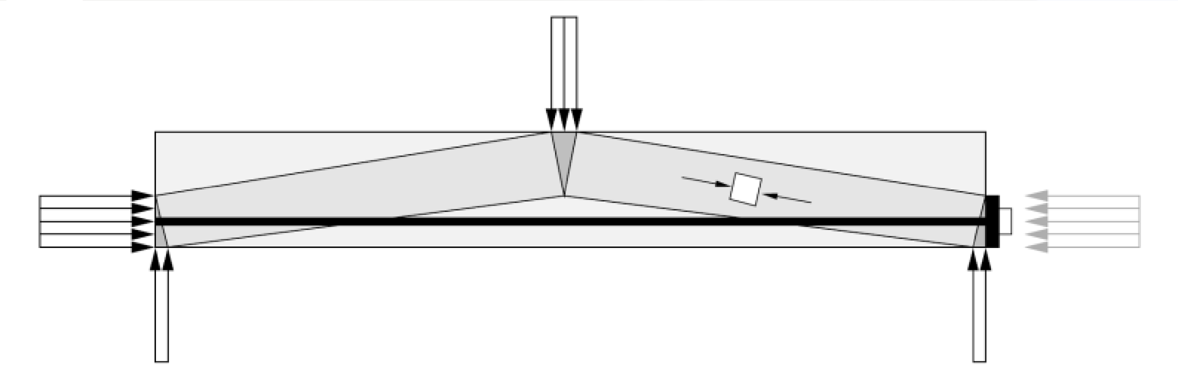


Ritter (1899)

2. Strut-and-tie and stress fields models



Drucker (1961)



2. Strut-and-tie and stress fields models

Investigaciones

Dimensionamiento y verificación del hormigón estructural mediante el método de los campos de tensiones

Dimensioning and check of structural concrete using the stress field method

Aurelio Muttoni
Prof. Dr., Ecole Polytechnique Fédérale de Lausanne, Suiza

Miguel Fernández Ruiz
Dr. Ecole Polytechnique Fédérale de Lausanne, Suiza

RESUMEN

Los campos de tensiones son una herramienta basada en el teorema del mínimo de la plasticidad que puede ser empleada tanto para el dimensionamiento como para la verificación de elementos o estructuras de hormigón armado y pretensado. En este artículo se resumen sus bases teóricas y de aplicación, proponiéndose finalmente un ejemplo del cual se analizan los resultados obtenidos.

SUMMARY

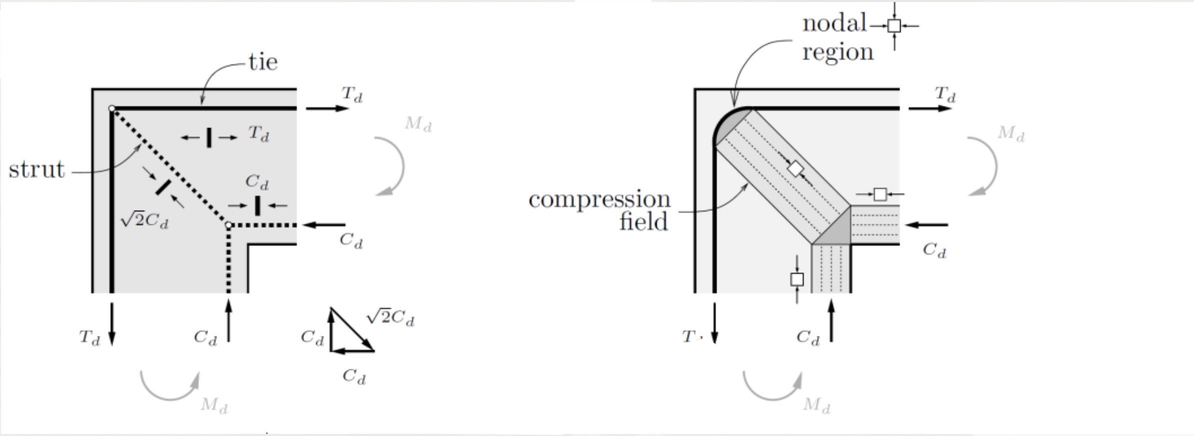
The stress field method is a tool based on the lower-bound theorem of plasticity. It can be applied for both the dimensioning of new structural elements and for the checking of existing ones. This paper reviews the theoretical principles of the method and its application to practical cases. An example is finally presented whose results are discussed.

1. INTRODUCCIÓN

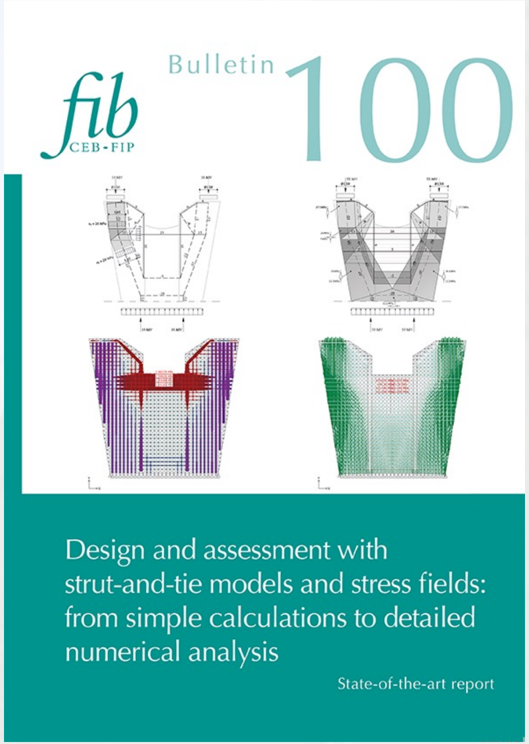
El método de los campos de tensiones es una herramienta fundada en la teoría de la plasticidad que permite el dimensionamiento y verificación de elementos y estructuras de hormigón armado y pretensado. Su aplicación no se restringe exclusivamente a este ámbito, pudiendo ser empleado además en el análisis de estructuras en acero estructural o bien de fábrica (ladrillo y piedra).

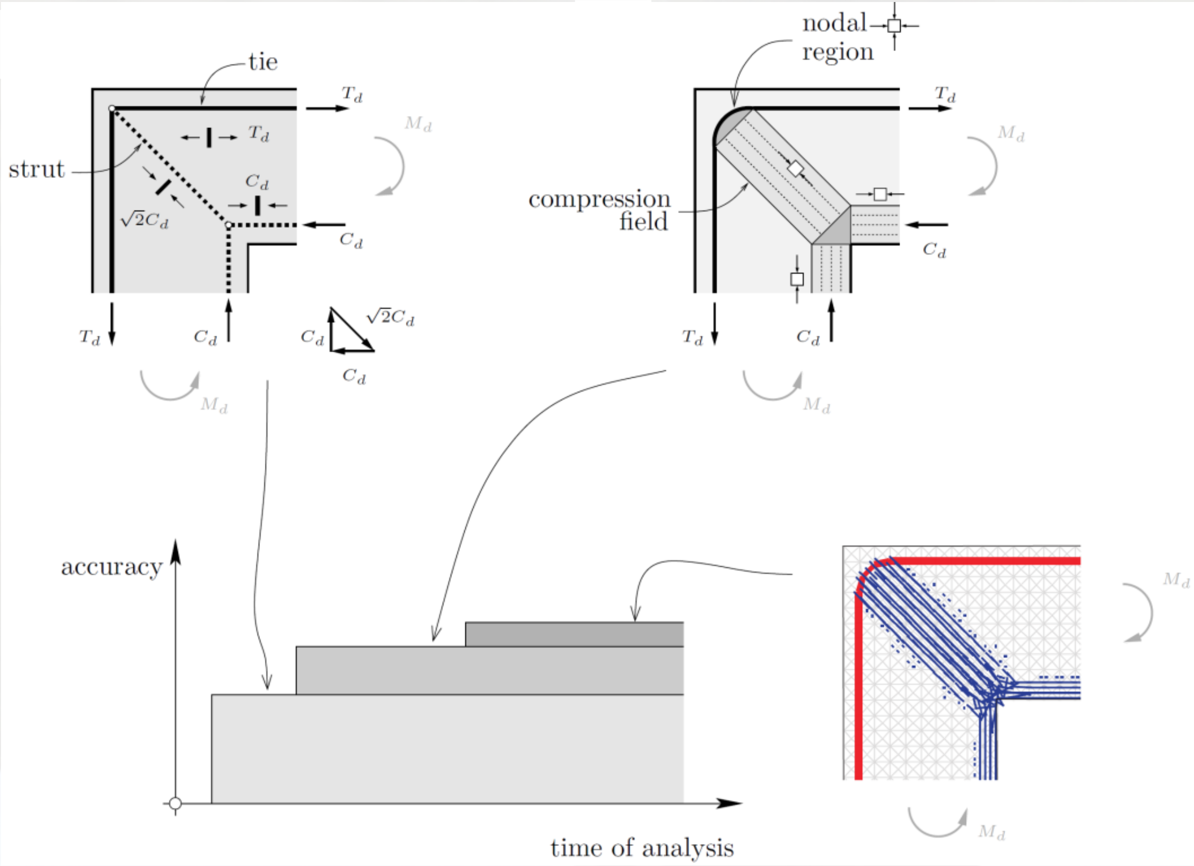
Existen además otros métodos de cálculo basados en la teoría de la plasticidad con aplicación al hormigón estructural. Entre ellos destaca principalmente el método de las bielas y los tirantes, el cual se basa en la denominada analogía de la celosía y que ha sido adoptado en la norma española EHE. El análisis del hormigón estructural mediante modelos de celosía comenzó en un principio como un desarrollo intuitivo del comportamiento del hormigón armado, sin base teórica sólida y sobre la base de resultados de ensayos. Ritter (1899) propuso por primera vez el análisis de una viga de hormigón armado mediante su idealización como una celosía. El desarrollo de modelos de celosía continuó posteriormente con Mörsch (1908) y su evolución ha seguido hasta nuestros días. En esta línea, Schlaich (1982, 1987) completando diferentes aspectos teóricos y prácticos de la analogía de la celosía, formalizó un método de análisis y comprobación de estructuras, el cual ha sido ampliamente difundido y empleado bajo el nombre de método de las bielas y los tirantes.

Los campos de tensiones nacen en cambio como una aplicación directa de la teoría de la plasticidad. El primer planteamiento de un campo de tensiones completo para un elemento estructural se debe a Drucker (1961) quien propuso un campo de tensiones lícito para una viga bajo carga puntual y bajo carga distribuida. El desarrollo posterior de la aplicación de los campos de tensiones al estudio del hormigón estructural se produjo fundamentalmente en la ETHZ (Zürich) de la mano del profesor Thürlimann, continuando su evolución hasta nuestros días con diversos trabajos sobre el tema (Müller 1978, Marti, 1980, Muttoni 1989). Otro centro destacado en el desarrollo de esta técnica ha sido la universidad de



2. Strut-and-tie and stress fields models

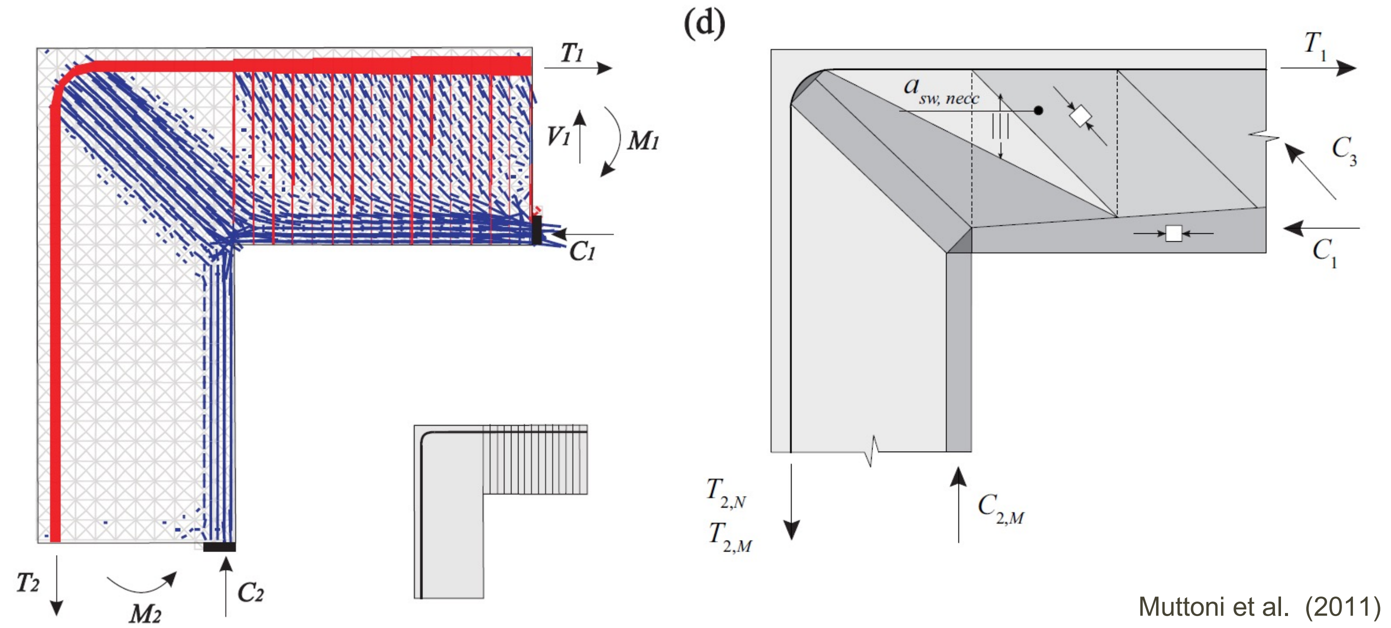




3. Compression fields and struts

Action:

$$\sigma_{cd} = \frac{|F_{cd}|}{b_c \cdot t}$$



Muttoni et al. (2011)

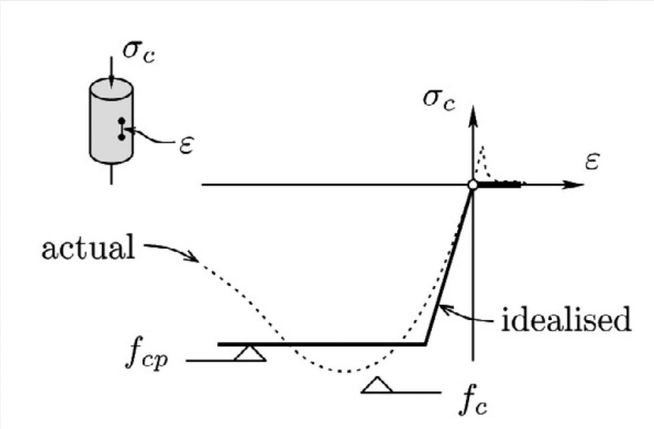
3. Compression fields and struts

Action:

$$\sigma_{cd} = \frac{|F_{cd}|}{b_c \cdot t}$$

Resistance:

$$\sigma_{cd} \leq \boxed{\nu} \cdot \boxed{f_{cd}}$$

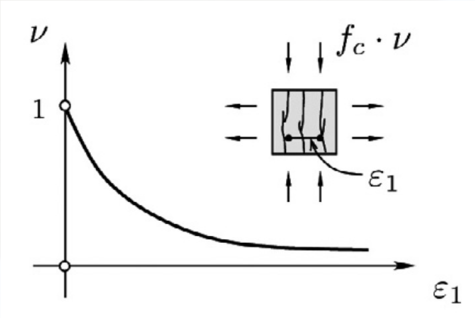


Plastic strength

(Muttoni, 1990)

$$f_{cd} = \eta_{cc} \cdot k_{tc} \frac{f_{ck}}{\gamma_C}$$

$$\eta_{cc} = \left(\frac{f_{ck,ref}}{f_{ck}}\right)^{\frac{1}{3}} \leq 1,0$$



Compression softening

(Robinson and Démorieux, 1962)

(Vecchio and Collins, 1986)

3. Compression fields and struts

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Resistance:

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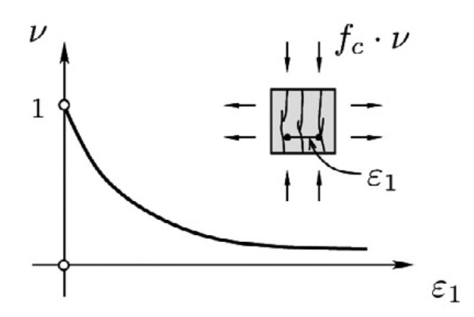
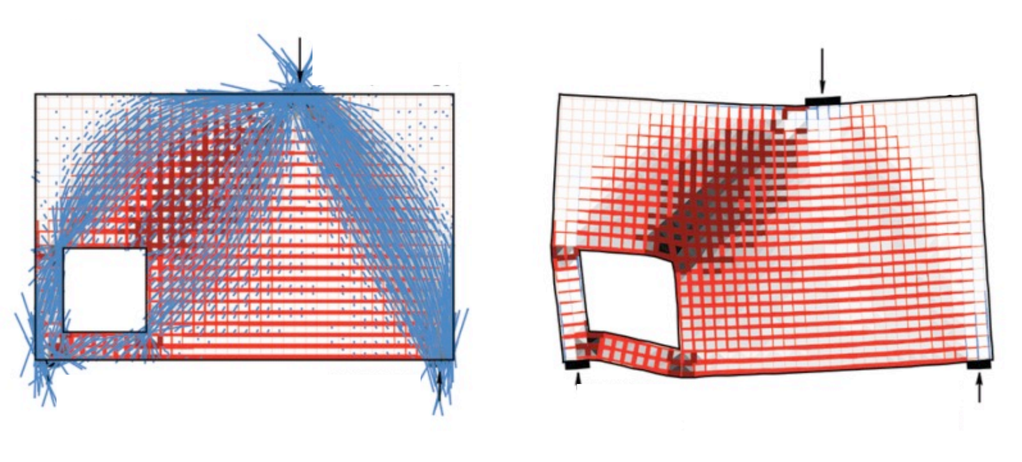


$$\nu = \frac{1}{1{,}0 + 110\varepsilon_1} \leq 1{,}0$$



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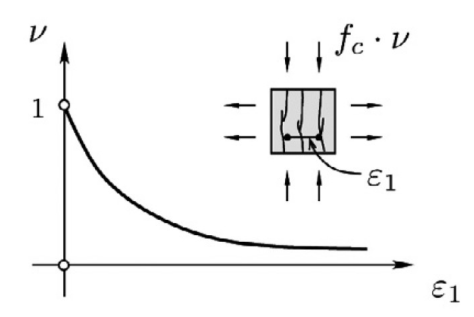

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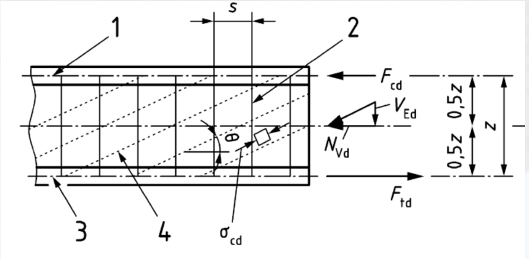
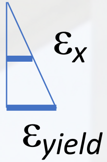
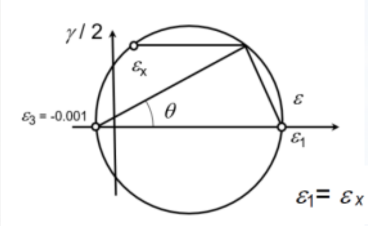




$$\varepsilon_x \approx \varepsilon_{yield} / 2 \approx 0.001$$

$$\nu = \frac{1}{1{,}11 + 0{,}22 \cdot \cot^2\theta_{cs}}$$



$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 0.001) \cdot \cot^2\theta$$

(but ε_x might have other values for panels with for instance uniform strain level)

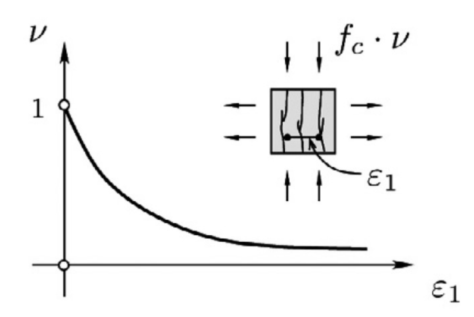
3. Compression fields and struts

Action:

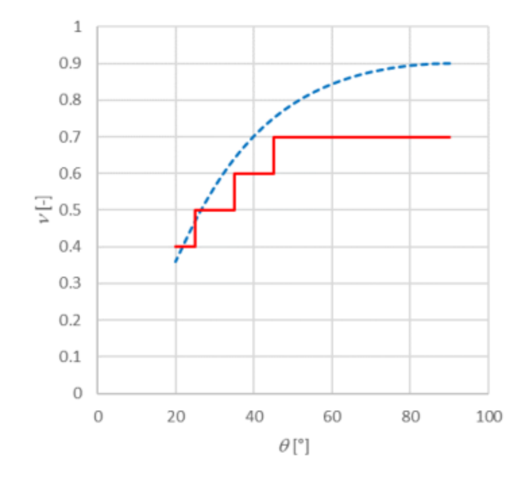
$$\sigma_{cd} = \frac{|F_{cd}|}{b_c \cdot t}$$

Resistance:

$$\sigma_{cd} \leq \boxed{\nu} \cdot f_{cd}$$



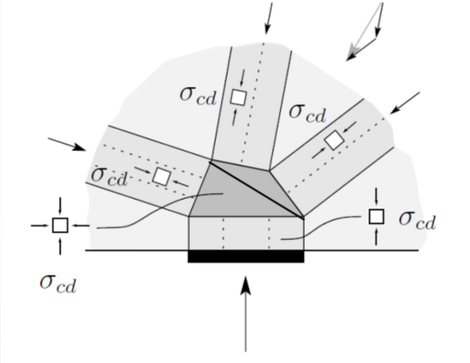
$$\nu = \frac{1}{1{,}0 + 110\varepsilon_1} \leq 1{,}0$$



$$\nu = \frac{1}{1{,}11 + 0{,}22 \cdot \cot^2\theta_{cs}}$$

4. Nodal regions

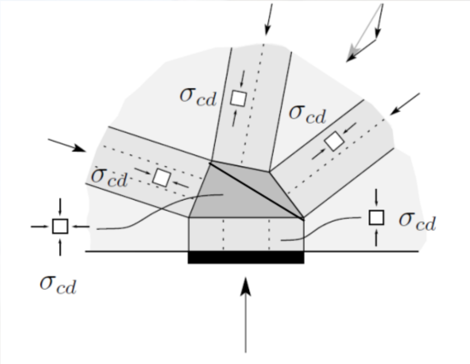
CCC



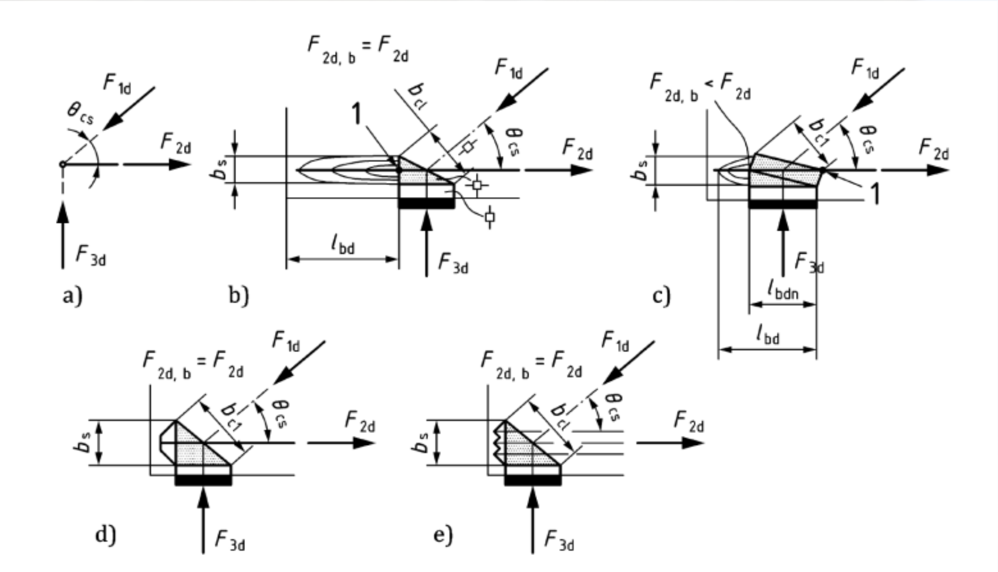
No need for detailed checks, ν is governed by strut conditions

4. Nodal regions

CCC

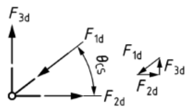


CCT

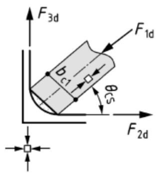


4. Nodal regions

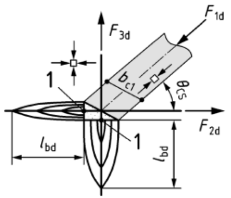
CTT



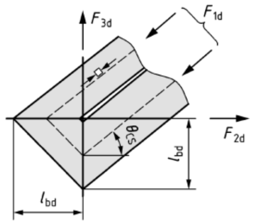
a) strut-and-tie model and equilibrium of forces



b) strut-and-tie model and stress field for the deviation force of a bent bar



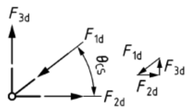
c) strut-and-tie model and stress field for anchorages outside the node



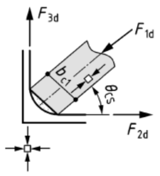
d) strut-and-tie model and stress field for anchorages inside the node with deviations occurring due to bond

4. Nodal regions

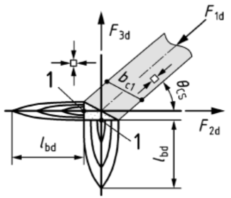
CTT



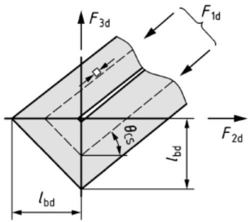
a) strut-and-tie model and equilibrium of forces



b) strut-and-tie model and stress field for the deviation force of a bent bar



c) strut-and-tie model and stress field for anchorages outside the node



d) strut-and-tie model and stress field for anchorages inside the node with deviations occurring due to bond

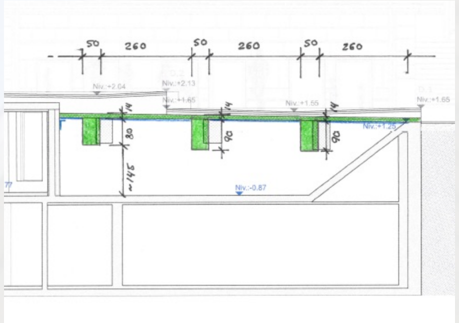
TTT

TTT nodes, where only ties reach the node. TTT nodes may be used only if consistent values of the strength reduction factor ν in the node are adopted accounting for anchorage, detailing and strains according to 8.5.2(5).

5. Example of application

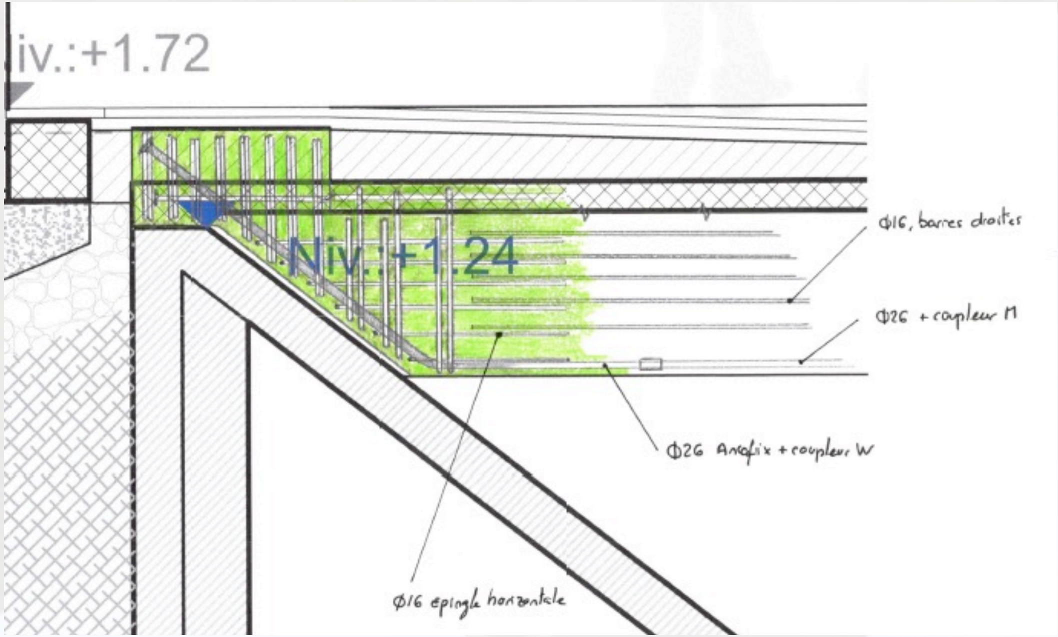
- Existing school at Lausanne (Switzerland)
- Refurbishment works, including a new courtyard
- Engineer: Antonio Garcia (Muttoni et Fernández, ingénieurs conseils SA)

5. Example of application

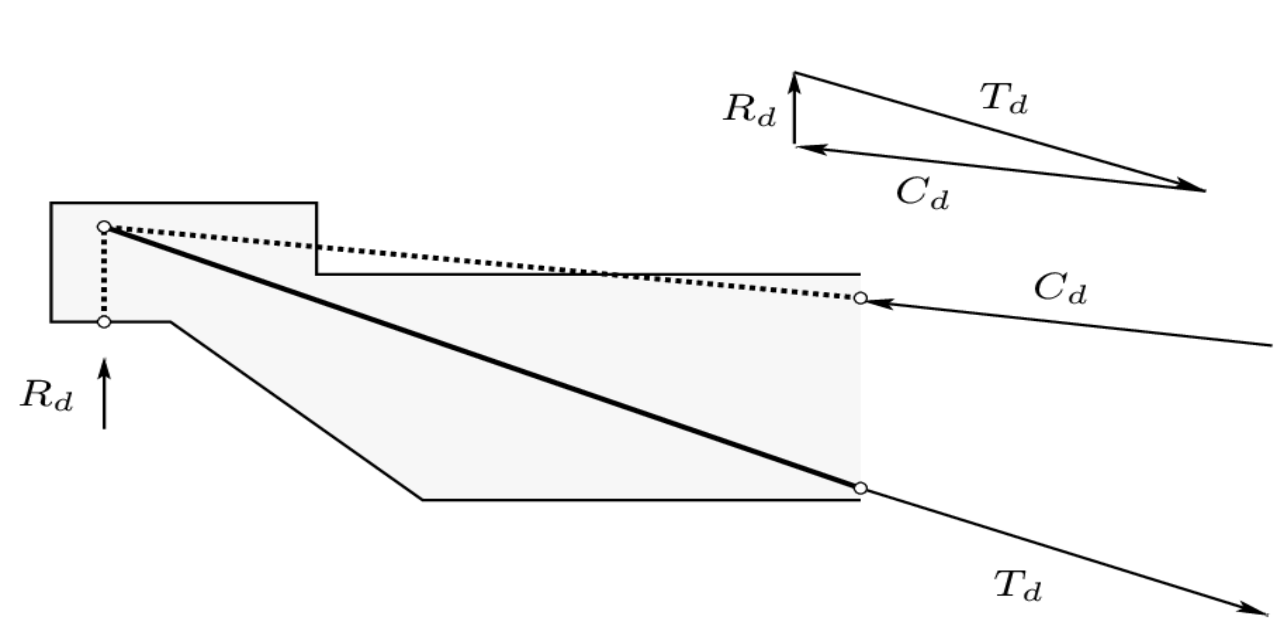


5. Example of application

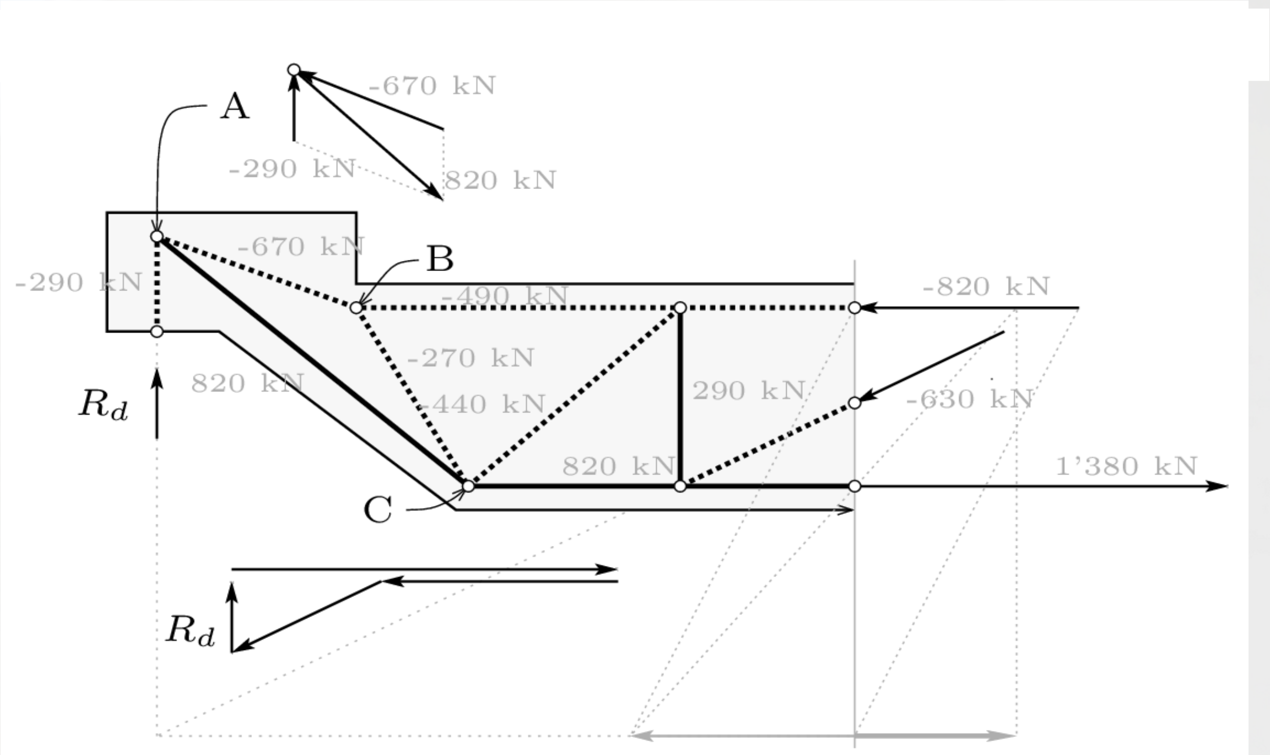
- Span length L = 12.1 m
- Main girders b = 50 cm
- Unusual shape of the support
- Heavily reinforced in bending
- How to approach a suitable detailing for the support region?



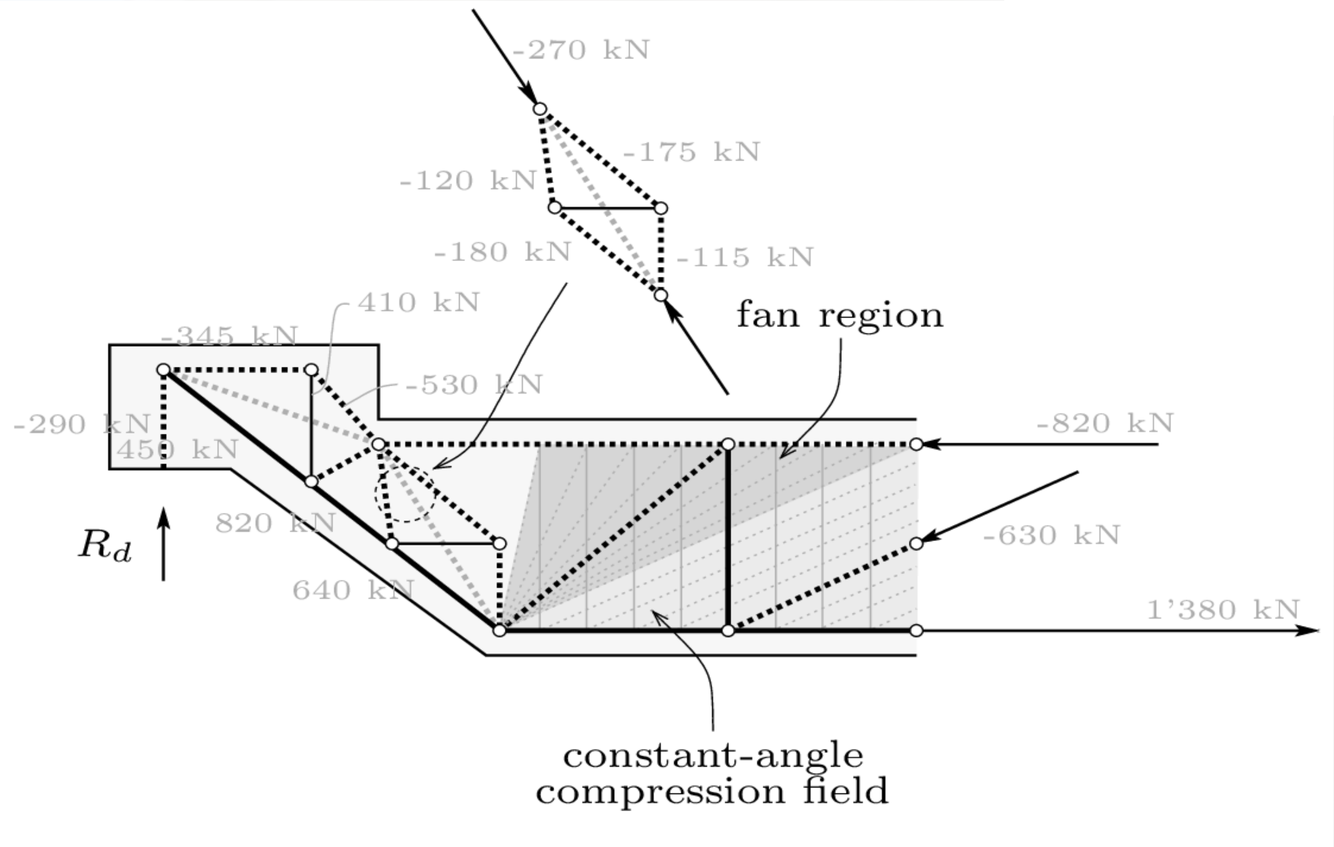
5. Example of application



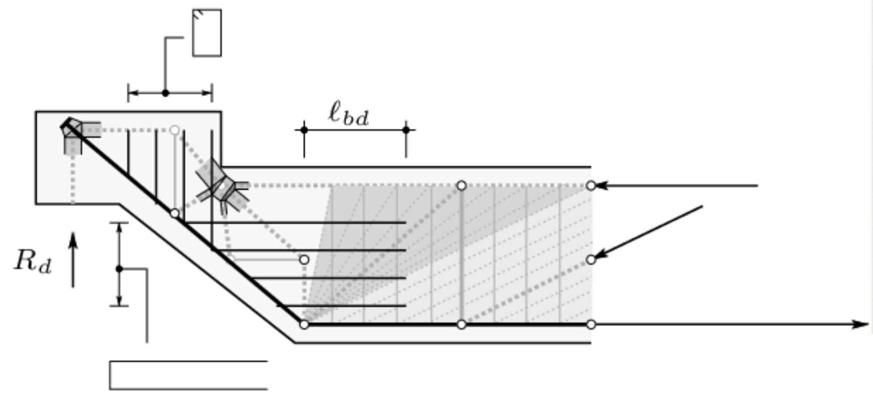
5. Example of application

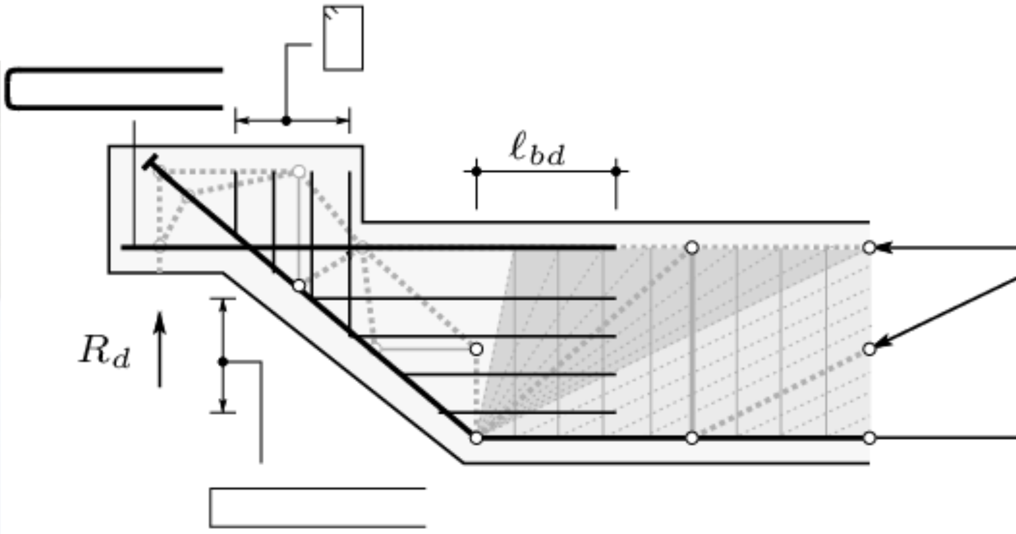


5. Example of application

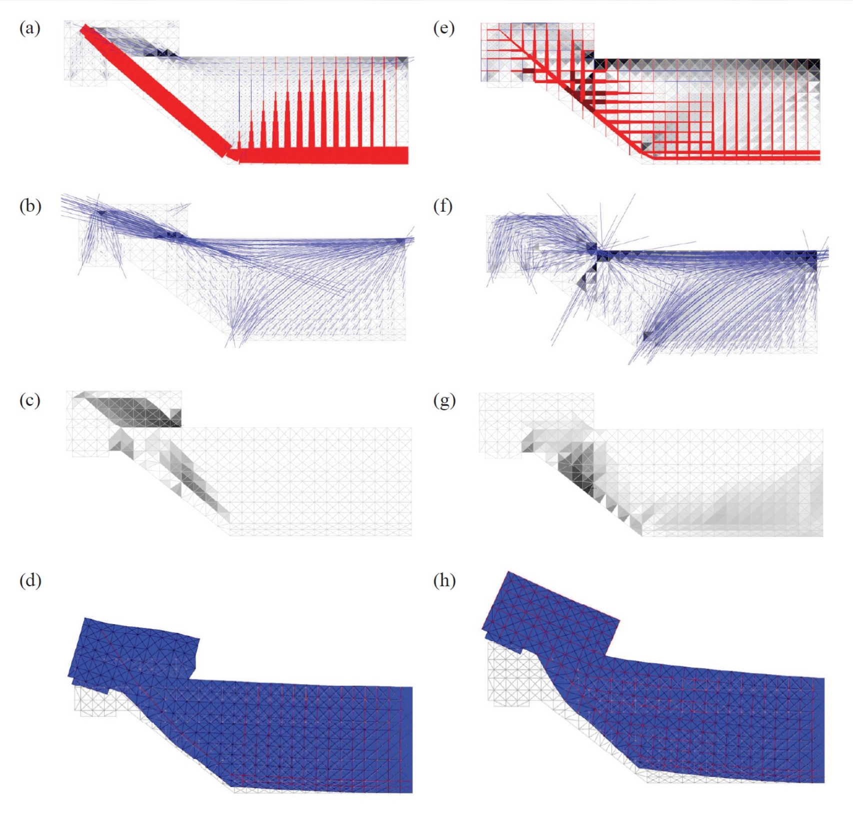


5. Example of application





5. Example of application

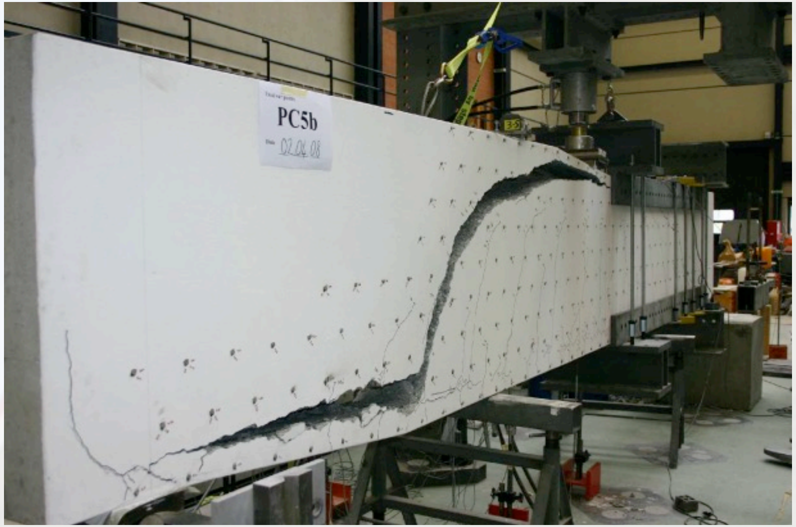


Thank you for your attention

Miguel Fernández Ruiz

2. Strut-and-tie and stress fields models

- Minimum reinforcement amount for crack control



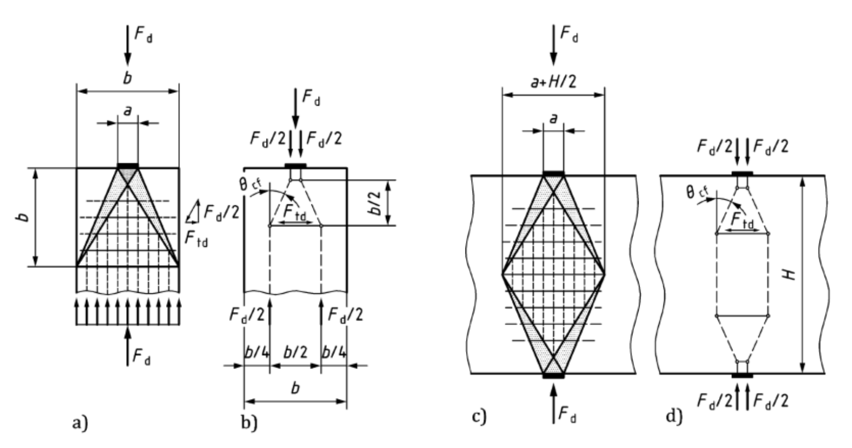
Campana et al. (2008)

Rupf et al. (2012)

- Reinforcement: ductility class B or C

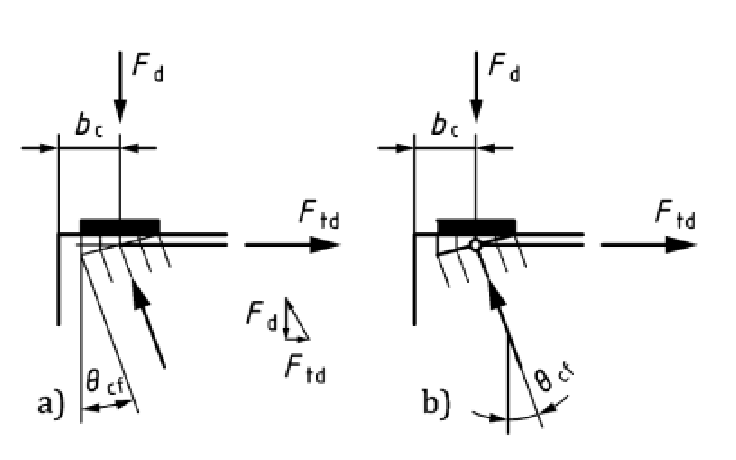
5. Special cases

Spreading of concentrated forces



$$\tan\theta_{cf} = \frac{1 - a/b}{2}$$

Forces at edges of walls



$$\tan\theta_{cf} \geq 1/4$$

Contents

1. <Name of Item 1>

LEVEL 1:

- LEVEL 2
  - LEVEL 3

LEVEL 1:

- LEVEL 2
  - LEVEL 3

LEVEL 1:

  - ...